Hoofdstuk 9: Exponentiële en logaritmische functies

9.1 Logaritmische en exponentiële vergelijkingen

Opgave 1:

a. y_2 en y_3

b. y_2 en y_3

c. y_1 en y_3

Opgave 2:

a. ${}^2\log 6 + {}^2\log 10 = {}^2\log 60$

b. ${}^3\log 30 - {}^3\log 6 = {}^3\log 5$

c. $2 \cdot {}^5\log 3 + {}^5\log \frac{1}{2} = {}^5\log 3^2 + {}^5\log \frac{1}{2} = {}^5\log 9 + {}^5\log \frac{1}{2} = {}^5\log 4\frac{1}{2}$

d. ${}^{\frac{1}{2}}\log 15 - 4 \cdot {}^{\frac{1}{2}}\log 3 = {}^{\frac{1}{2}}\log 15 - {}^{\frac{1}{2}}\log 3^4 = {}^{\frac{1}{2}}\log 15 - {}^{\frac{1}{2}}\log 81 = {}^{\frac{1}{2}}\log \frac{15}{81} = {}^{\frac{1}{2}}\log \frac{5}{27}$

e. $-2 \cdot {}^4\log 6 + {}^4\log 12 = {}^4\log 6^{-2} + {}^4\log 12 = {}^4\log \frac{1}{36} + {}^4\log 12 = {}^4\log \frac{12}{36} = {}^4\log \frac{1}{3}$

f. $\log 50 - 2 \cdot \log 5 = \log 50 - \log 5^2 = \log 50 - \log 25 = \log 2$

Opgave 3:

a. $4 + {}^2\log 3 = {}^2\log 2^4 + {}^2\log 3 = {}^2\log 16 + {}^2\log 3 = {}^2\log 48$

b. $3 - {}^{\frac{1}{2}}\log 10 = {}^{\frac{1}{2}}\log (\frac{1}{2})^3 - {}^{\frac{1}{2}}\log 10 = {}^{\frac{1}{2}}\log \frac{1}{8} - {}^{\frac{1}{2}}\log 10 = {}^{\frac{1}{2}}\log \frac{1}{80}$

c. $2 - \log 5 = \log 10^2 - \log 5 = \log 100 - \log 5 = \log 20$

d. ${}^2\log 12 - {}^3\log 9 = {}^2\log 12 - 2 = {}^2\log 12 - {}^2\log 2^2 = {}^2\log 12 - {}^2\log 4 = {}^2\log 3$

e. $\frac{1}{2} \cdot {}^3\log 16 + {}^{\frac{1}{2}}\log 8 = {}^3\log 16^{\frac{1}{2}} + -3 = {}^3\log 4 - {}^3\log 3^3 = {}^3\log 4 - {}^3\log 27 = {}^3\log \frac{4}{27}$

f. $\log 500 - {}^5\log 125 = \log 500 - 3 = \log 500 - \log 10^3 = \log 500 - \log 1000 = \log \frac{1}{2}$

Opgave 4:

a. ${}^3\log 6 + {}^3\log 1\frac{1}{2} = {}^3\log 9 = 2$

b. ${}^5\log 2 - {}^5\log 50 = {}^5\log \frac{2}{50} = {}^5\log \frac{1}{25} = -2$

c. ${}^2\log 27 + 3 \cdot {}^2\log \frac{1}{6} = {}^2\log 27 + {}^2\log (\frac{1}{6})^3 = {}^2\log 27 + {}^2\log \frac{1}{216} = {}^2\log \frac{27}{216} = {}^2\log \frac{1}{8} = -3$

d. $2 \cdot {}^4\log 6 - 2 \cdot {}^4\log 3 = {}^4\log 6^2 - {}^4\log 3^2 = {}^4\log 36 - {}^4\log 9 = {}^4\log 4 = 1$

Opgave 5:

a. $g^{{}^g\log a - {}^g\log b} = \dfrac{g^{{}^g\log a}}{g^{{}^g\log b}} = \dfrac{a}{b} = g^{{}^g\log \frac{a}{b}}$

b. $g^{n \cdot {}^g\log a} = \left(g^{{}^g\log a}\right)^n = a^n = g^{{}^g\log a^n}$

Opgave 6:

a. $3 + {}^2\log 3 = {}^2\log 2^3 + {}^2\log 3 = {}^2\log 8 + {}^2\log 3 = {}^2\log 24$

b. ${}^2\log(x+1) = 3 + {}^2\log 3$

${}^2\log(x+1) = {}^2\log 24$

$x + 1 = 24$

$x = 23$

Opgave 7:

a. $^5\log x = 3 \cdot {}^5\log 2 - 2 \cdot {}^5\log 3$

$^5\log x = {}^5\log 2^3 - {}^5\log 3^2$

$^5\log x = {}^5\log 8 - {}^5\log 9$

$^5\log x = {}^5\log \frac{8}{9}$

$x = \frac{8}{9}$

b. $^2\log x = 4 - {}^2\log 3$

$^2\log x = {}^2\log 4^2 - {}^2\log 3$

$^2\log x = {}^2\log 16 - {}^2\log 3$

$^2\log x = {}^2\log \frac{16}{3}$

$x = \frac{16}{3} = 5\frac{1}{3}$

c. $^2\log(x+3) = 3 + {}^2\log x$

$^2\log(x+3) = {}^2\log 2^3 + {}^2\log x$

$^2\log(x+3) = {}^2\log 8 + {}^2\log x$

$^2\log(x+3) = {}^2\log 8x$

$x + 3 = 8x$

$-7x = -3$

$x = \frac{3}{7}$

d. $^3\log 2x = 1 + {}^3\log(x+1)$

$^3\log 2x = {}^3\log 3 + {}^3\log(x+1)$

$^3\log 2x = {}^3\log 3(x+1)$

$2x = 3(x+1)$

$2x = 3x + 3$

$-x = 3$

$x = -3$ vervalt

dus geen oplossingen

Opgave 8:

a. $5 \cdot \log x = 5 - \log 3125$

$\log x = 1 - \frac{1}{5} \cdot \log 3125$

$\log x = 1 - \log 3125^{\frac{1}{5}}$

$\log x = \log 10 - \log 5$

$\log x = \log 2$

$x = 2$

b. $^{\frac{1}{2}}\log(2x-1) = 2 + {}^{\frac{1}{2}}\log(x+2)$

$^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{2})^2 + {}^{\frac{1}{2}}\log(x+2)$

$^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log \frac{1}{4} + {}^{\frac{1}{2}}\log(x+2)$

$^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log \frac{1}{4}(x+2)$

$2x-1=\frac{1}{4}(x+2)$

$2x-1=\frac{1}{4}x+\frac{1}{2}$

$1\frac{3}{4}x=1\frac{1}{2}$

$x=\frac{6}{7}$

c. $^3\log(x+2)=1-{}^3\log x$

$^3\log(x+2)={}^3\log 3-{}^3\log x$

$^3\log(x+2)+{}^3\log x={}^3\log 3$

$^3\log x(x+2)={}^3\log 3$

$x(x+2)=3$

$x^2+2x=3$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3$ (vervalt) $\quad\vee\quad x=1$

dus $x=1$

d. $2\cdot{}^3\log x+1={}^3\log(5x-2)$

$^3\log x^2+{}^3\log 3={}^3\log(5x-2)$

$^3\log 3x^2={}^3\log(5x-2)$

$3x^2=5x-2$

$3x^2-5x+2=0$

$x=\dfrac{5\pm\sqrt{1}}{6}=\dfrac{5\pm 1}{6}$

$x=1\quad\vee\quad x=\frac{2}{3}$

Opgave 9:

a. $^5\log x=2+\frac{1}{2}\cdot{}^5\log 3$

$^5\log x={}^5\log 25+{}^5\log 3^{\frac{1}{2}}$

$^5\log x={}^5\log 25+{}^5\log\sqrt{3}$

$^5\log x={}^5\log 25\sqrt{3}$

$x=25\sqrt{3}$

b. $^3\log(x+4)+1=2\cdot{}^3\log(x-2)$

$^3\log(x+4)+{}^3\log 3={}^3\log(x-2)^2$

$^3\log 3(x+4)={}^3\log(x-2)^2$

$3(x+4)=(x-2)^2$

$3x+12=x^2-4x+4$

$-x^2+7x+8=0$

$x^2-7x-8=0$

$(x+1)(x-8)=0$

$x=-1$ vervalt $\quad\vee\quad x=8$

dus $x=8$

c. $^2\log 2x-{}^2\log(x+3)={}^2\log x-2$

$^{2}\log 2x + 2 = {}^{2}\log(x+3) + {}^{2}\log x$

$^{2}\log 2x + {}^{2}\log 4 = {}^{2}\log x(x+3)$

$^{2}\log 8x = {}^{2}\log(x^2 + 3x)$

$8x = x^2 + 3x$

$-x^2 + 5x = 0$

$-x(x-5) = 0$

$x = 0$ vervalt $\vee$ $x = 5$

dus $x = 5$

d. $^{3}\log x = 2 - {}^{3}\log(x-1)$

$^{3}\log x + {}^{3}\log(x-1) = 2$

$^{3}\log x(x-1) = {}^{3}\log 9$

$x(x-1) = 9$

$x^2 - x = 9$

$x^2 - x - 9 = 0$

$x = \frac{1 \pm \sqrt{37}}{2}$

$x = \frac{1}{2} + \frac{1}{2}\sqrt{37}$ $\vee$ $x = \frac{1}{2} - \frac{1}{2}\sqrt{37}$ vervalt

dus $x = \frac{1}{2} + \frac{1}{2}\sqrt{37}$

Opgave 10:

a. $p^2 - 2p - 8 = 0$

$(p+2)(p-4) = 0$

$p = -2$ $\vee$ $p = 4$

b. $^{2}\log x = -2$ $\vee$ $^{2}\log x = 4$

$x = 2^{-2} = \frac{1}{4}$ $\vee$ $x = 2^4 = 16$

Opgave 11:

$^{3}\log 4 = \frac{\log 4}{\log 3} = 1{,}262$

$^{\frac{1}{2}}\log 3 = \frac{\log 3}{\log \frac{1}{2}} = -1{,}585$

Opgave 12:

a. $^{3}\log(3x-5) + {}^{\frac{1}{3}}\log(x-1) = 0$

$^{3}\log(3x-5) - {}^{3}\log(x-1) = 0$

$^{3}\log(3x-5) = {}^{3}\log(x-1)$

$3x - 5 = x - 1$

$2x = 4$

$x = 2$

b. $^{5}\log 3x + 2 \cdot {}^{\frac{1}{5}}\log x = 0$

$^{5}\log 3x + 2 \cdot - {}^{5}\log x = 0$

$^5\log 3x = 2 \cdot {}^5\log x$

$^5\log 3x = {}^5\log x^2$

$3x = x^2$

$-x^2 + 3x = 0$

$-x(x-3) = 0$

$x = 0$ vervalt $\vee$ $x = 3$

dus $x = 3$

c. $2x \cdot {}^{\frac{1}{3}}\log(3x+5) = {}^{\frac{1}{3}}\log(3x+5)$

$2x = 1 \quad \vee \quad {}^{\frac{1}{3}}\log(3x+5) = 0$

$x = \frac{1}{2} \quad \vee \quad 3x + 5 = (\frac{1}{3})^0$

$x = \frac{1}{2} \quad \vee \quad 3x + 5 = 1$

$x = \frac{1}{2} \quad \vee \quad 3x = -4$

$x = \frac{1}{2} \quad \vee \quad x = -1\frac{1}{3}$

d. $^2\log^2 x = 2 \cdot {}^2\log x + 3$

stel $p = {}^2\log x$ dan $p^2 = 2p + 3$

$p^2 - 2p - 3 = 0$

$(p+1)(p-3) = 0$

$p = -1 \quad \vee \quad p = 3$

$^2\log x = -1 \quad \vee \quad {}^2\log x = 3$

$x = 2^{-1} = \frac{1}{2} \quad \vee \quad x = 2^3 = 8$

Opgave 13:

a. $-2 \cdot {}^{\frac{1}{2}}\log x = 2 + {}^2\log(3-x)$

$-2 \cdot - {}^2\log x = {}^2\log 4 + {}^2\log(3-x)$

$^2\log x^2 = {}^2\log 4(3-x)$

$x^2 = 4(3-x)$

$x^2 = 12 - 4x$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6$ vervalt $\vee$ $x = 2$

dus $x = 2$

b. $^9\log 2x = {}^3\log(x-4)$

$\frac{{}^3\log 2x}{{}^3\log 9} = {}^3\log(x-4)$

$\frac{{}^3\log 2x}{2} = {}^3\log(x-4)$

$^3\log 2x = 2 \cdot {}^3\log(x-4)$

$^3\log 2x = {}^3\log(x-4)^2$

$2x = (x-4)^2$

$2x = x^2 - 8x + 16$

$-x^2+10x-16=0$
$x^2-10x+16=0$
$(x-2)(x-8)=0$
$x=2$ vervalt $\vee$ $x=8$
dus $x=8$

c. $4x\cdot{}^4\log(2x-1)+3\cdot{}^4\log(2x-1)=0$
$(4x+3)\cdot{}^4\log(2x-1)=0$
$4x+3=0 \vee {}^4\log(2x-1)=0$
$4x=-3 \vee 2x-1=4^0$
$x=-\frac{3}{4} \vee 2x-1=1$
$x=-\frac{3}{4} \vee 2x=2$
$x=-\frac{3}{4}$ vervalt $\vee$ $x=1$
dus $x=1$

d. ${}^{\frac{1}{2}}\log^2(x+2)+3\cdot{}^{\frac{1}{2}}\log(x+2)=0$
stel $p={}^{\frac{1}{2}}\log(x+2)$ dan $p^2+3p=0$
$p(p+3)=0$
$p=0 \vee p=-3$
${}^{\frac{1}{2}}\log(x+2)=0 \vee {}^{\frac{1}{2}}\log(x+2)=-3$
$x+2=(\frac{1}{2})^0 \vee x+2=(\frac{1}{2})^{-3}$
$x+2=1 \vee x+2=8$
$x=-1 \vee x=6$

Opgave 14:

a. $3x\cdot{}^2\log(x+1)={}^{\frac{1}{2}}\log(x+1)$
$3x\cdot{}^2\log(x+1)=-{}^2\log(x+1)$
$3x\cdot{}^2\log(x+1)+{}^2\log(x+1)=0$
$(3x+1)\cdot{}^2\log(x+1)=0$
$3x+1=0 \vee {}^2\log(x+1)=0$
$3x=-1 \vee x+1=2^0$
$x=-\frac{1}{3} \vee x+1=1$
$x=-\frac{1}{3} \vee x=0$

b. $x^2\cdot{}^5\log(2x+1)+9\cdot{}^{\frac{1}{5}}\log(2x+1)=0$
$x^2\cdot{}^5\log(2x+1)+9\cdot-{}^5\log(2x+1)=0$
$(x^2-9)\cdot{}^5\log(2x+1)=0$
$x^2-9=0 \vee {}^5\log(2x+1)=0$
$x^2=9 \vee 2x+1=5^0$
$x=3 \vee x=-3$ vervalt $\vee$ $2x+1=1$
$x=3 \vee 2x=0$
$x=3 \vee x=0$

c. $2\cdot{}^{3}\log^2 x + 2 = 5\cdot{}^{3}\log x$

stel $p = {}^{3}\log x$ dan $2p^2 + 2 = 5p$

$2p^2 - 5p + 2 = 0$

$p = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$

$p = 2 \quad \vee \quad p = \frac{1}{2}$

${}^{3}\log x = 2 \quad \vee \quad {}^{3}\log x = \frac{1}{2}$

$x = 3^2 = 9 \quad \vee \quad x = 3^{\frac{1}{2}} = \sqrt{3}$

d. ${}^{5}\log^2 x + 3\cdot{}^{\frac{1}{5}}\log x + 2 = 0$

${}^{5}\log^2 x + 3\cdot - {}^{5}\log x + 2 = 0$

stel $p = {}^{5}\log x$ dan $p^2 - 3p + 2 = 0$

$(p-1)(p-2) = 0$

$p = 1 \quad \vee \quad p = 2$

${}^{5}\log x = 1 \quad \vee \quad {}^{5}\log x = 2$

$x = 5^1 = 5 \quad \vee \quad x = 5^2 = 25$

Opgave 15:

7 kun je niet als macht van 2 schrijven

Opgave 16:

a. $p^2 + 2p = 8$

$p^2 + 2p - 8 = 0$

$(p+4)(p-2) = 0$

$p = -4 \quad \vee \quad p = 2$

b. $2^x = -4 \quad \vee \quad 2^x = 2$

k.n. $\qquad x = 1$

Opgave 17:

a. $3^x - 2 = 8\cdot(\frac{1}{3})^x$

$3^x - 2 = 8\cdot\frac{1}{3^x}$

stel $p = 3^x$ dan $p - 2 = 8\cdot\frac{1}{p}$

$p^2 - 2p = 8$

$p^2 - 2p - 8 = 0$

$(p+2)(p-4) = 0$

$p = -2 \quad \vee \quad p = 4$

$3^x = -2 \quad \vee \quad 3^x = 4$

k.n. $\qquad x = {}^{3}\log 4$

b. $2^x = 6 - 5\cdot(\frac{1}{2})^x$

$2^x = 6 - 5 \cdot \frac{1}{2^x}$

stel $p = 2^x$ dan $p = 6 - 5 \cdot \frac{1}{p}$

$p^2 = 6p - 5$

$p^2 - 6p + 5 = 0$

$(p-1)(p-5) = 0$

$p = 1 \quad \vee \quad p = 5$

$2^x = 1 \quad \vee \quad 2^x = 5$

$x = 0 \quad \vee \quad x = {}^2\log 5$

c. $9^x = 4 + 3^{x+1}$

$(3^2)^x = 4 + 3^1 \cdot 3^x$

$(3^x)^2 = 4 + 3 \cdot 3^x$

Stel $p = 3^x$ dan $p^2 = 4 + 3p$

$p^2 - 3p - 4 = 0$

$(p+1)(p-4) = 0$

$p = -1 \quad \vee \quad p = 4$

$3^x = -1 \quad \vee \quad 3^x = 4$

k.n. $\qquad x = {}^3\log 4$

d. $2^x = 24 - 2^{2x-1}$

$2^x = 24 - 2^{-1} \cdot 2^{2x}$

$2^x = 24 - \frac{1}{2} \cdot (2^x)^2$

stel $p = 2^x$ dan $p = 24 - \frac{1}{2}p^2$

$\frac{1}{2}p^2 + p - 24 = 0$

$p^2 + 2p - 48 = 0$

$(p+8)(p-6) = 0$

$p = -8 \quad \vee \quad p = 6$

$2^x = -8 \quad \vee \quad 2^x = 6$

k.n. $\qquad x = {}^2\log 6$

Opgave 18:

a. $3^{2x-1} = 10$

$2x - 1 = {}^3\log 10$

$2x = 1 + {}^3\log 10$

$x = \frac{1 + {}^3\log 10}{2} = \frac{1}{2} + \frac{1}{2} \cdot {}^3\log 10 = 1{,}55$

b. $5 \cdot 4^{x-2} = 16$

$4^{x-2} = 3{,}2$

$x - 2 = {}^4\log 3{,}2$

$x + 2 + {}^4\log 3{,}2 = 2{,}84$

c. $9^x = 2\cdot 3^x + 6$

$(3^2)^x = 2\cdot 3^x + 6$

$(3^x)^2 = 2\cdot 3^x + 6$

stel $p = 3^x$ dan $p^2 = 2p + 6$

$p^2 - 2p - 6 = 0$

$p = \frac{2\pm\sqrt{28}}{2} = \frac{2\pm 2\sqrt{7}}{2} = 1\pm\sqrt{7}$

$3^x = 1+\sqrt{7} \quad \vee \quad 3^x = 1-\sqrt{7}$ (k.n.)

$x={}^3\log(1+\sqrt{7}) = 1{,}18$

d. $2^x + 2^{-x} = 3$

$2^x + \frac{1}{2^x} = 3$

stel $p = 2^x$ dan $p + \frac{1}{p} = 3$

$p^2 + 1 = 3p$

$p^2 - 3p + 1 = 0$

$p = \frac{3\pm\sqrt{5}}{2} = 1\tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{5}$

$2^x = 1\tfrac{1}{2} + \sqrt{5} \quad \vee \quad 2^x = 1\tfrac{1}{2} - \sqrt{5}$

$x={}^2\log(1\tfrac{1}{2}+\sqrt{5}) = 1{,}39 \quad \vee \quad x={}^2\log(1\tfrac{1}{2}-\tfrac{1}{2}\sqrt{5}) = -1{,}39$

Opgave 19:

a. $3^{x+2} + 3^x = 600$

$3^2\cdot 3^x + 1\cdot 3^x = 600$

$9\cdot 3^x + 1\cdot 3^x = 600$

$10\cdot 3^x = 600$

$3^x = 60$

$x={}^3\log 60$

b. $5^{x-1} + 5^{2x-1} = 4$

$5^{-1}\cdot 5^x + 5^{-1}\cdot 5^{2x} = 4$

$\tfrac{1}{5}\cdot 5^x + \tfrac{1}{5}\cdot (5^x)^2 = 4$

stel $p = 5^x$ dan $\tfrac{1}{5}p + \tfrac{1}{5}p^2 = 4$

$\tfrac{1}{5}p^2 + \tfrac{1}{5}p - 4 = 0$

$p^2 + p - 20 = 0$

$(p+5)(p-4) = 0$

$p = -5 \quad \vee \quad p = 4$

$5^x = -5 \quad \vee \quad 5^x = 4$

k.n. $\qquad x={}^5\log 4$

c. $3^x + 5\cdot(\tfrac{1}{3})^{x-2} = 18$

$3^x + 5\cdot(3^{-1})^{x-2} = 18$

$$3^x + 5 \cdot 3^{-x+2} = 18$$

$$3^x + 5 \cdot \frac{3^2}{3^x} = 18$$

$$3^x + \frac{45}{3^x} = 18$$

stel $p = 3^x$ dan $p + \dfrac{45}{p} = 18$

$$p^2 + 45 = 18p$$

$$p^2 - 18p + 45 = 0$$

$$(p-3)(p-15) = 0$$

$$p = 3 \quad \vee \quad p = 15$$

$$3^x = 3 \quad \vee \quad 3^x = 15$$

$$x = 1 \quad \vee \quad x = {}^3\log 15$$

d. $3^x + 2 \cdot (\tfrac{1}{3})^{x-2} = 1$

$$3^x + 2 \cdot (3^{-1})^{x-2} = 1$$

$$3^x + 2 \cdot 3^{-x+2} = 1$$

$$3^x + 2 \cdot \frac{3^2}{3^x} = 1$$

$$3^x + \frac{18}{3^x} = 1$$

stel $p = 3^x$ dan $p + \dfrac{18}{p} = 1$

$$p^2 + 18 = p$$

$$p^2 - p + 18 = 0$$

$$p = \frac{1 \pm \sqrt{-71}}{2} = \text{k.n.}$$

dus geen oplossingen

9.2 Grafieken van exponentiële en logaritmische functies

Opgave 20:

a. $T(-3,0)$

b. $f(x) = 2^{x+3} = 2^3 \cdot 2^x = 8 \cdot 2^x$

Opgave 21:

a. $V_{y-as,\frac{1}{8}}$

b. $f(x) = {}^2\log 8x = {}^2\log 8 + {}^2\log x = 3 + {}^2\log x$

Opgave 22:

a. $y = 2^x \xrightarrow{T(5,0)} y = 2^{x-5}$

$y = 2^{x-5} = 2^{-5} \cdot 2^x = \frac{1}{32} \cdot 2^x$ dus $V_{x-as,\frac{1}{32}}$

b. $y = 4^x \xrightarrow{V_{x-as,2}} y = 2 \cdot 4^x$

$y = 2 \cdot 4^x = 4^{\frac{1}{2}} \cdot 4^x = 4^{x+\frac{1}{2}}$ dus $T(-\frac{1}{2},0)$

c. $y = {}^2\log x \xrightarrow{V_{y-as,\frac{1}{32}}} y = {}^2\log 32x$

$y = {}^2\log 32x = {}^2\log 32 + {}^2\log x = 5 + {}^2\log x$ dus $T(0,5)$

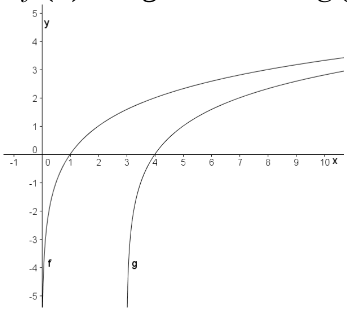
d. $y = {}^4\log x \xrightarrow{T(0,\frac{1}{2})} y = \frac{1}{2} + {}^4\log x$

$y = \frac{1}{2} + {}^4\log x = {}^4\log 4^{\frac{1}{2}} + {}^4\log x = {}^4\log 2 + {}^4\log x = {}^4\log 2x$ dus $V_{y-as,\frac{1}{2}}$

Opgave 23:

a. $f(x) = {}^2\log x \xrightarrow{T(3,0)} g(x) = {}^2\log(x-3)$

b.



c. Nee, de verticale asymptoot van de grafiek van *f* is de lijn $x = 0$ dus dit zou na vermenigvuldiging ook de verticale asymptoot van de grafiek van *g* moeten zijn, maar dat is de lijn $x = 3$. Dus er bestaat geen vermenigvuldiging ten opzichte van de *y*-as. Er is ook geen verticale translatie mogelijk want voor $x = 1$ bestaat *f* wel maar *g* niet.

d. $g(x) = {}^2\log(x-3) \xrightarrow{V_{y-as,\frac{1}{4}}} h(x) = {}^2\log(4x-3)$

$h(x) = {}^2\log(4x-3) = {}^2\log 4(x-\frac{3}{4}) = {}^2\log 4 + {}^2\log(x-\frac{3}{4}) = 2 + {}^2\log(x-\frac{3}{4})$

dus $p = -\frac{3}{4}$ en $q = 2$

Opgave 24:

a. $V_{x-as,4}$

b. $g(x) = 4 \cdot (\frac{1}{2})^x = 2^2 \cdot (\frac{1}{2})^x = ((2^{-1})^{-1})^2 \cdot (\frac{1}{2})^x = (\frac{1}{2})^{-2} \cdot (\frac{1}{2})^x = (\frac{1}{2})^{x-2}$ dus $T(2,0)$

c. $h(x) = 4^x = (2^2)^x = 2^{2x} = ((\tfrac{1}{2})^{-1})^{2x} = (\tfrac{1}{2})^{-2x}$ dus $V_{y-as,-\frac{1}{2}}$

d. $h(x) = 4^x = (2^2)^x = 2^{2x} = ((\tfrac{1}{2})^{-1})^{2x} = (\tfrac{1}{2})^{-2x}$ dus $V_{x-as,\frac{1}{4}}$ en $V_{y-as,-\frac{1}{2}}$

e. $g(x) = 4 \cdot (\tfrac{1}{2})^x \xrightarrow{T(3,4)} j(x) = 4 + 4 \cdot (\tfrac{1}{2})^{x-3}$

$j(x) = 4 + 4 \cdot (\tfrac{1}{2})^{x-3} = 4 + 4 \cdot (\tfrac{1}{2})^{-3} \cdot (\tfrac{1}{2})^x = 4 + 4 \cdot 8 \cdot (\tfrac{1}{2})^x = 4 + 32 \cdot (\tfrac{1}{2})^x$

dus $a = 32$ en $b = 4$

Opgave 25:

$AB = g(1) - f(1) = 6 - \tfrac{1}{2} = 5\tfrac{1}{2}$

Opgave 26:

a. $y = g({}^2\log 6\tfrac{2}{5}) = 8 - 2^{{}^2\log 6\frac{2}{5}} = 8 - 6\tfrac{2}{5} = 1\tfrac{3}{5}$

b. de twee horizontale asymptoten liggen op een afstand 8 van elkaar, dus alleen $f(x) - g(x) = 10$ heeft een oplossing

c. $0 < a < 8$

Opgave 27:

a. $3^{x-1} - 2 = 4 - 3^x$

$3^{x-1} + 3^x = 6$

$3^{-1} \cdot 3^x + 3^x = 6$

$\tfrac{1}{3} \cdot 3^x + 3^x = 6$

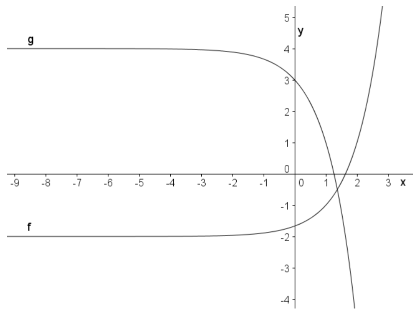
$\tfrac{4}{3} \cdot 3^x = 6$

$3^x = 4\tfrac{1}{2}$

$x = {}^3\log 4\tfrac{1}{2}$

$y = -\tfrac{1}{2}$

dus $A = ({}^3\log 4\tfrac{1}{2}, -\tfrac{1}{2})$



b. $f(x) - g(x) = 6$

$3^{x-1} - 2 - (4 - 3^x) = 6$

$3^{x-1} - 2 - 4 + 3^x = 6$

$3^{x-1} + 3^x = 12$

$3^{-1} \cdot 3^x + 3^x = 12$

$\tfrac{1}{3} \cdot 3^x + 3^x = 12$

$\tfrac{4}{3} \cdot 3^x = 12$

$3^x = 9$

$x = 2$ dus $p = 2$

Opgave 28:

a. $1 - 3x > 0$

$-3x > -1$

$x < \tfrac{1}{3}$ dus $D_f = \langle \leftarrow, \tfrac{1}{3} \rangle$

$x + 5 > 0$

$x > -5$ dus $D_g = \langle -5, \rightarrow \rangle$

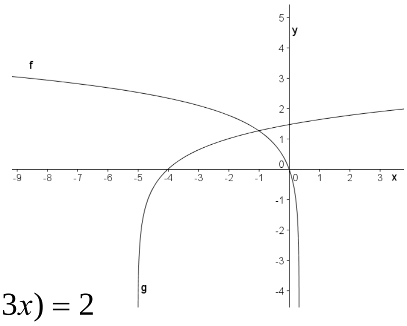
b. $\,^{3}\log(1-3x) \leq \,^{3}\log(x+5)$

$1-3x \leq x+5$

$-4x \leq 4$

$x \geq -1$

dus $-1 \leq x < \frac{1}{3}$



c. $f(x)-g(x)=2 \quad \vee \quad g(x)-f(x)=2$

$\,^{3}\log(1-3x) - \,^{3}\log(x+5) = 2 \quad \vee \quad \,^{3}\log(x+5) - \,^{3}\log(1-3x) = 2$

$\,^{3}\log\frac{1-3x}{x+5} = \,^{3}\log 9 \quad \vee \quad \,^{3}\log\frac{x+5}{1-3x} = \,^{3}\log 9$

$\frac{1-3x}{x+5} = 9 \quad \vee \quad \frac{x+5}{1-3x} = 9$

$9(x+5) = 1-3x \quad \vee \quad x+5 = 9(1-3x)$

$9x+45 = 1-3x \quad \vee \quad x+5 = 9-27x$

$12x = -44 \quad \vee \quad 28x = 4$

$x = -3\frac{2}{3} \quad \vee \quad x = \frac{1}{7}$

dus $p = -3\frac{2}{3} \quad \vee \quad p = \frac{1}{7}$

Opgave 29:

a. $(\frac{3}{2})^{x+2} = 3\cdot(\frac{2}{3})^{x} + 3$

$(\frac{3}{2})^{x+2} = 3\cdot((\frac{3}{2})^{-1})^{x} + 3$

$(\frac{3}{2})^{x+2} = 3\cdot(\frac{3}{2})^{-x} + 3$

$(\frac{3}{2})^{2}\cdot(\frac{3}{2})^{x} = 3\cdot\frac{1}{(\frac{3}{2})^{x}} + 3$

stel $p = (\frac{3}{2})^{x}$ dan $\frac{9}{4}p = 3\cdot\frac{1}{p} + 3$

$\frac{9}{4}p^{2} = 3+3p$

$\frac{9}{4}p^{2} - 3p - 3 = 0$

$9p^{2} - 12p - 12 = 0$

$p = \frac{12 \pm \sqrt{576}}{18} = \frac{12 \pm 24}{18}$

$p = 2 \quad \vee \quad p = -\frac{2}{3}$

$(\frac{3}{2})^{x} = 2 \quad \vee \quad (\frac{3}{2})^{x} = -\frac{2}{3}$

$x = \,^{\frac{3}{2}}\log 2$ k.n.

$y = 4\frac{1}{2}$

b. $f(x)-g(x)=4 \quad \vee \quad g(x)-f(x)=4$

$y_1 = (\frac{3}{2})^{x+2} - (3\cdot(\frac{2}{3})^{x} + 3)$ | $y_1 = 3\cdot(\frac{2}{3})^{x} + 3 - (\frac{3}{2})^{x+2}$

$y_2 = 4$ | $y_2 = 4$

intersect geeft $x = 3{,}085$ | intersect geeft $x = -0{,}117$

dus $p = 3{,}085 \quad \vee \quad p = -0{,}117$

Opgave 30:

a. $x_B = x_A + AB = p + 6$

b. A en B liggen op een horizontale lijn dus $y_A = y_B$

$f(x_A) = g(x_B)$

$f(p) = g(p+6)$

c. $q = y_A = f(p)$

Opgave 31:

$f(x) = g(x + 1\tfrac{1}{8}) \quad \vee \quad g(x) = f(x + 1\tfrac{1}{8})$

${}^{\frac{1}{2}}\log 2x = 2 + {}^{\frac{1}{2}}\log(x + 1\tfrac{1}{8} + 2) \quad \vee \quad 2 + {}^{\frac{1}{2}}\log(x+2) = {}^{\frac{1}{2}}\log 2(x + 1\tfrac{1}{8})$

${}^{\frac{1}{2}}\log 2x = {}^{\frac{1}{2}}\log \tfrac{1}{4} + {}^{\frac{1}{2}}\log(x + 3\tfrac{1}{8}) \quad \vee \quad {}^{\frac{1}{2}}\log \tfrac{1}{4} + {}^{\frac{1}{2}}\log(x+2) = {}^{\frac{1}{2}}\log 2(x + 1\tfrac{1}{8})$

${}^{\frac{1}{2}}\log 2x = {}^{\frac{1}{2}}\log \tfrac{1}{4}(x + 3\tfrac{1}{8}) \quad \vee \quad {}^{\frac{1}{2}}\log \tfrac{1}{4}(x+2) = {}^{\frac{1}{2}}\log 2(x + 1\tfrac{1}{8})$

$2x = \tfrac{1}{4}(x + 3\tfrac{1}{8}) \quad \vee \quad \tfrac{1}{4}(x+2) = 2(x + 1\tfrac{1}{8})$

$8x = x + 3\tfrac{1}{8} \quad \vee \quad x + 2 = 8(x + 1\tfrac{1}{8})$

$7x = 3\tfrac{1}{8} \quad \vee \quad x + 2 = 8x + 9$

$x = \tfrac{25}{56} \quad \vee \quad -7x = 7$

$y = {}^{\frac{1}{2}}\log \tfrac{25}{28} \quad \vee \quad x = -1$

$y = 2$

dus $q = {}^{\frac{1}{2}}\log \tfrac{25}{28} \quad \vee \quad q = 2$

Opgave 32:

a. de twee verticale asymptoten hebben een afstand van 2, dus alleen rechts van het snijpunt is er een horizontaal lijnstuk met lengte 2

b. $0 < a < 2$

Opgave 33:

$f(x) = g(x+2) \quad \vee \quad g(x) = f(x+2)$

$2^{x-2} = 8 - 2^{x+2} \quad \vee \quad 8 - 2^x = 2^{x+2-2}$

$2^{x-2} + 2^{x+2} = 8 \quad \vee \quad 8 = 2^x + 2^x$

$2^{-2} \cdot 2^x + 2^2 \cdot 2^x = 8 \quad \vee \quad 8 = 2 \cdot 2^x$

$\tfrac{1}{4} \cdot 2^x + 4 \cdot 2^x = 8 \quad \vee \quad 4 = 2^x$

$4\tfrac{1}{4} \cdot 2^x = 8 \quad \vee \quad x = 2$

$2^x = \tfrac{32}{17} \qquad y = 4$

$x = {}^2\log \tfrac{32}{17}$

$y = \tfrac{8}{17}$

dus $q = \tfrac{8}{17} \quad \vee \quad q = 4$

Opgave 34:

a. ${}^4\log(x^2 - 1) \leq {}^2\log(x+3)$ voorwaarde: $(x < -1 \quad \vee \quad x > 1) \quad \wedge \quad x > -3$

$\dfrac{{}^2\log(x^2-1)}{{}^2\log 4} = {}^2\log(x+3)$ $\qquad D_f = \langle\leftarrow, -1\rangle$ en $\langle 1, \rightarrow\rangle$

$\dfrac{{}^2\log(x^2-1)}{2} = {}^2\log(x+3)$ $\qquad D_g = \langle -3, \rightarrow\rangle$

${}^2\log(x^2-1) = 2 \cdot {}^2\log(x+3)$

${}^2\log(x^2-1) = {}^2\log(x+3)^2$

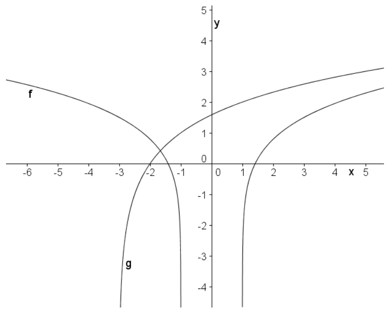
$x^2 - 1 = (x+3)^2$

$x^2 - 1 = x^2 + 6x + 9$

$-6x = 10$

$x = -\frac{5}{3}$

$-\frac{5}{3} \le x < -1 \quad \vee \quad x > 1$



b. $f(x) - g(x) = \frac{1}{2} \quad \vee \quad g(x) - f(x) = \frac{1}{2}$

${}^4\log(x^2-1) - {}^2\log(x+3) = \frac{1}{2} \quad \vee \quad {}^2\log(x+3) - {}^4\log(x^2-1) = \frac{1}{2}$

$\frac{1}{2} \cdot {}^2\log(x^2-1) - {}^2\log(x+3) = \frac{1}{2} \quad \vee \quad {}^2\log(x+3) - \frac{1}{2} \cdot {}^2\log(x^2-1) = \frac{1}{2}$

${}^2\log(x^2-1) - 2 \cdot {}^2\log(x+3) = 1 \quad \vee \quad 2 \cdot {}^2\log(x+3) - {}^2\log(x^2-1) = 1$

${}^2\log(x^2-1) - {}^2\log(x+3)^2 = 1 \quad \vee \quad {}^2\log(x+3)^2 - {}^2\log(x^2-1) = 1$

${}^2\log\dfrac{x^2-1}{(x+3)^2} = {}^2\log 2 \quad \vee \quad {}^2\log\dfrac{(x+3)^2}{x^2-1} = {}^2\log 2$

$\dfrac{x^2-1}{(x+3)^2} = 2 \quad \vee \quad \dfrac{(x+3)^2}{x^2-1} = 2$

$2(x+3)^2 = x^2 - 1 \quad \vee \quad 2(x^2-1) = (x+3)^2$

$2x^2 + 12x + 18 = x^2 - 1 \quad \vee \quad 2x^2 - 2 = x^2 + 6x + 9$

$x^2 + 12x + 19 = 0 \quad \vee \quad x^2 - 6x - 11 = 0$

$x = \dfrac{-12 \pm \sqrt{68}}{2} = -6 \pm \sqrt{17} \quad \vee \quad x = \dfrac{6 \pm \sqrt{80}}{2} = 3 \pm 2\sqrt{5}$

$x = -6 - \sqrt{17}$ (vervalt) $\quad \vee \quad x = -6 + \sqrt{17} \quad \vee \quad x = 3 - 2\sqrt{5} \quad \vee \quad x = 3 + 2\sqrt{5}$

dus $p = -6 + \sqrt{17} \quad \vee \quad p = 3 - 2\sqrt{5} \quad \vee \quad p = 3 + 2\sqrt{5}$

c. $f(x) = g(x+1) \quad \vee \quad g(x) = f(x+1)$

${}^4\log(x^2-1) = {}^2\log(x+1+3) \quad \vee \quad {}^2\log(x+3) = {}^4\log((x+1)^2 - 1)$

$\frac{1}{2} \cdot {}^2\log(x^2-1) = {}^2\log(x+4) \quad \vee \quad {}^2\log(x+3) = \frac{1}{2} \cdot {}^2\log(x^2+2x)$

${}^2\log(x^2-1)^{\frac{1}{2}} = {}^2\log(x+4) \quad \vee \quad {}^2\log(x+3) = {}^2\log(x^2+2x)^{\frac{1}{2}}$

$(x^2-1)^{\frac{1}{2}} = x + 4 \quad \vee \quad x + 3 = (x^2+2x)^{\frac{1}{2}}$

$x^2 - 1 = (x+4)^2 \quad \vee \quad (x+3)^2 = x^2 + 2x$

$x^2 - 1 = x^2 + 8x + 16 \quad \vee \quad x^2 + 6x + 9 = x^2 + 2x$

$-8x = 17 \quad \vee \quad 4x = -9$

$x = -\frac{17}{8} \quad \vee \quad x = -2\frac{1}{4}$

$y = {}^2\log\frac{15}{8} \qquad y = {}^2\log\frac{3}{4}$

$y = {}^2\log 15 - {}^2\log 8 = {}^2\log 15 - 3 \qquad y = {}^2\log 3 - {}^2\log 4 = {}^2\log 3 - 2$

$y = -3 + {}^2\log 15 \qquad y = -2 + {}^2\log 3$

Opgave 35:

a. $AB = p$

$AB : BC = 1 : 2 = p : 2p$

dus $BC = 2p$

$AC = AB + BC = p + 2p = 3p$

b. $y_B = 2^p$

$y_C = g(3p) = 2^{3p-3}$

$y_B = y_C$ dus $2^p = 2^{3p-3}$

$p = 3p - 3$

$-2p = -3$

$p = 1\frac{1}{2}$

c. $q = y_B = 2^p = 2^{1\frac{1}{2}} = 2\sqrt{2}$

Opgave 36:

$x_B = p$ dan $x_C = 2p$

$f(p) = f(2p)$

$6p \cdot 2^{-p} = 6 \cdot 2p \cdot 2^{-2p}$

$6p \cdot 2^{-p} = 12p \cdot 2^{-2p}$

$6p = 0 \quad \vee \quad 2^{-p} = 2 \cdot 2^{-2p}$

$p = 0 \quad \vee \quad 2^{-p} = 2^{1-2p}$

$-p = 1 - 2p$

$p = 1$

$q = f(1) = 6 \cdot 2^{-1} = 6 \cdot \frac{1}{2} = 3$

Opgave 37:

a. $x_B = p$ dan $x_C = 3p$

$y_B = y_C$

$f(p) = g(3p)$

${}^2\log p = {}^2\log(3p - 3)$

$p = 3p - 3$

$-2p = -3$

$p = 1\frac{1}{2}$

$q = f(1\frac{1}{2}) = {}^2\log 1\frac{1}{2}$

b. $y_F = 2 \cdot y_E$

$f(r) = 2 \cdot g(r)$

${}^2\log r = 2 \cdot {}^2\log(r - 3)$

${}^2\log r = {}^2\log(r - 3)^2$

$r = (r - 3)^2$

$r = r^2 - 6r + 9$

$r^2 - 7r + 9 = 0$

$r = \frac{7 \pm \sqrt{13}}{2}$

$r = \frac{7 - \sqrt{13}}{2} = 1{,}697$ (vervalt) $\quad \vee \quad r = \frac{7 + \sqrt{13}}{2} = 5{,}303$

dus $r = 5{,}303$

Opgave 38:

$x_B = p$ dan $x_C = 3p$

$y_B = y_C$

$f(p) = f(3p)$

$8p \cdot (\frac{1}{3})^p = 8 \cdot 3p \cdot (\frac{1}{3})^{3p}$

$8p \cdot (\frac{1}{3})^p = 24p \cdot (\frac{1}{3})^{3p}$

$8p = 0 \quad \vee \quad (\frac{1}{3})^p = 3 \cdot (\frac{1}{3})^{3p}$

$p = 0 \quad \vee \quad (\frac{1}{3})^p = (\frac{1}{3})^{-1} \cdot (\frac{1}{3})^{3p}$

k.n. $\quad \vee \quad (\frac{1}{3})^p = (\frac{1}{3})^{3p-1}$

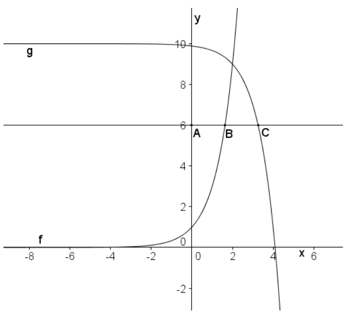
$p = 3p - 1$

$-2p = -1$

$p = \frac{1}{2}$

$y = f(\frac{1}{2}) = 4 \cdot (\frac{1}{3})^{\frac{1}{2}} = 4 \cdot \sqrt{\frac{1}{3}} = 4 \cdot \frac{1}{3}\sqrt{3} = \frac{4}{3}\sqrt{3}$

Opgave 39:



a. $x_B = p$ dan $x_C = 2p$

$f(p) = g(2p)$

$3^p = 10 - 3^{2p-2}$

$3^p + 3^{2p-2} = 10$

$3^{-2} \cdot 3^{2p} + 3^p - 10 = 0$

$\frac{1}{9} \cdot (3^p)^2 + 3^p - 10 = 0$

neem $r = 3^p$ dan $\frac{1}{9}r^2 + r - 10 = 0$

$r^2 + 9r - 90 = 0$

$(r + 15)(r - 6) = 0$

$r = -15 \quad \vee \quad r = 6$

$3^p = -15 \quad \vee \quad 3^p = 6$

k.n. $\qquad p = {}^3\log 6$

$q = f({}^3\log 6) = 3^{{}^3\log 6} = 6$

b. $y_F = 2 \cdot y_E$

$g(r) = 2 \cdot f(r)$

$10 - 3^{r-2} = 2 \cdot 3^r$

$3^{r-2} + 2 \cdot 3^r - 10 = 0$

$3^{-2} \cdot 3^r + 2 \cdot 3^r - 10 = 0$

$\frac{1}{9} \cdot 3^r + 2 \cdot 3^r - 10 = 0$

$2\frac{1}{9} \cdot 3^r = 10$

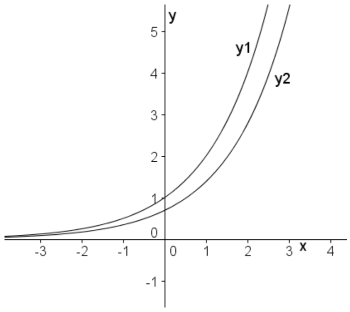
$3^r = 4\frac{14}{19}$

$r = {}^3\log 4\frac{14}{19}$

9.3 Het grondtal *e*

Opgave 40:

a.



b. $c = 0{,}6931$

c. $c = 1{,}0986$

Opgave 41:

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h \to 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim_{h \to 0} \frac{2^x \cdot (2^h - 1)}{h}$

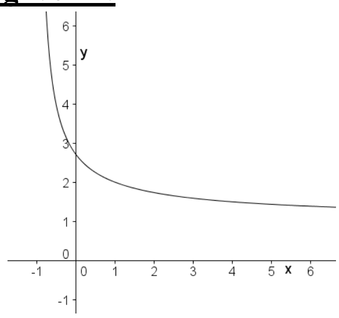
$= \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x$

b. $f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 2^0 = \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 1 = \lim_{h \to 0} \frac{2^h - 1}{h}$

c. $f'(x) = \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x = f'(0) \cdot 2^x$

Opgave 42:

a.



b. $\frac{1}{x}$ bestaat niet voor $x = 0$

c. $x = 0{,}01$ $y_1 = 2{,}7048$

$x = 0{,}001$ $y_1 = 2{,}7169$

$x = 0{,}0001$ $y_1 = 2{,}7181$

$x = 0{,}00001$ $y_1 = 2{,}7181$

d. $a = 2{,}718$

Opgave 43:

a. $2e^2 - e^2 = e^2$

b. $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$

c. $5e^2 \cdot 3e^3 = 15e^5$

d. $\frac{12e^6}{4e^2} = 3e^4$

e. $e^{5x} \cdot e^x = e^{6x}$

f. $e^x \cdot e^2 = e^{x+2}$

g. $5e^x - 3e^x = 2e^x$

h. $e^x \cdot (e^2 + 1) = e^{x+2} + e^x$

i. $e^x \cdot (e^x + 1) = e^{2x} + e^x$

j. $(e^x + 1)^2 = e^{2x} + 2e^x + 1$

k. $(e^{3x} + 3)^2 = e^{6x} + 6e^{3x} + 9$

l. $\frac{6e^{2x} - e^x}{e^x} = \frac{6e^{2x}}{e^x} - \frac{e^x}{e^x} = 6e^x - 1$

Opgave 44:

a. $(2 + 3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$

b. $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$

c. $\frac{e^{2x} - 4}{e^x - 2} = \frac{(e^x - 2)(e^x + 2)}{e^x - 2} = e^x + 2$

Opgave 45:

a. $(2x + 4)e^x = 0$

$2x + 4 = 0 \quad \vee \quad e^x = 0$

$2x = -4$ k.n.

$x = -2$

b. $x^2 e^x = 3xe^x$

$x^2 e^x - 3xe^x = 0$

$x(x - 3)e^x = 0$

$x = 0 \quad \vee \quad x = 3 \quad \vee \quad e^x = 0$

$x = 0 \quad \vee \quad x = 3$ k.n.

c. $x^2 e^x = e^x$

$x^2 e^x - e^x = 0$

$(x^2 - 1)e^x = 0$

$x^2 = 1 \quad \vee \quad e^x = 0$ (k.n.)

$x = 1 \quad \vee \quad x = -1$

d. $e^{3x} - e^x = 0$

$e^{3x} = e^x$

$3x = x$

$2x = 0$

$x = 0$

e. $e^{4x} - 1 = 0$

$e^{4x} = 1$

$e^{4x} = e^0$

$4x = 0$

$x = 0$

f. $e^x \cdot e^x = e^6$

$e^{2x} = e^6$

$2x = 6$

$x = 3$

Opgave 46:

a. $e^x + e^x = 2e^6$

$2e^x = 2e^6$

$e^x = e^6$

$x = 6$

b. $\dfrac{e^{5x}}{e^x} = e$

$e^{4x} = e^1$

$4x = 1$

$x = \frac{1}{4}$

c. $2xe^x + e^x = 0$

$(2x+1)e^x = 0$

$2x = -1 \quad \vee \quad e^x = 0$

$x = -\frac{1}{2}$ k.n.

d. $e^{x+2} - \sqrt{e} = 0$

$e^{x+2} = \sqrt{e}$

$e^{x+2} = e^{\frac{1}{2}}$

$x + 2 = \frac{1}{2}$

$x = -1\frac{1}{2}$

e. $e^{2x} + e^x = 2$

$(e^x)^2 + e^x - 2 = 0$

neem $p = e^x$ dan $p^2 + p - 2 = 0$

$(p+2)(p-1) = 0$

$p = -2 \quad \vee \quad p = 1$

$e^x = -2 \quad \vee \quad e^x = 1$

k.n. $x = 0$

f. $e^{6x} + 1 = 2e^{3x}$

$(e^{3x})^2 - 2e^{3x} + 1 = 0$

neem $p = e^{3x}$ dan $p^2 - 2p + 1 = 0$

$(p-1)^2 = 0$

$p = 1$

$e^{3x} = 1$

$e^{3x} = e^0$

$3x = 0$

$x = 0$

Opgave 47:

a. $f(x) = xe^x$

$f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$

b. $g(x) = \dfrac{e^x}{x+1}$ `

$g'(x) = \dfrac{(x+1)\cdot e^x - e^x \cdot 1}{(x+1)^2} = \dfrac{xe^x + e^x - e^x}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$

Opgave 48:

a. $f(x) = e^x + 2$

$f'(x) = e^x$

b. $f(x) = 2e^x + \dfrac{1}{x} = 2e^x + x^{-1}$

$f'(x) = 2e^x - x^{-2} = 2e^x - \dfrac{1}{x^2}$

c. $f(x) = xe^x + 4$

$f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$

d. $f(x) = \dfrac{x}{e^x}$

$f'(x) = \dfrac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \dfrac{1-x}{e^x}$

e. $f(x) = \dfrac{2e^x}{x-1}$

$f'(x) = \dfrac{(x-1)\cdot 2e^x - 2e^x \cdot 1}{(x-1)^2} = \dfrac{2xe^x - 2e^x - 2e^x}{(x-1)^2} = \dfrac{(2x-4)e^x}{(x-1)^2}$

f. $f(x) = (2x-4)e^x$

$f'(x) = 2 \cdot e^x + (2x-4)\cdot e^x = 2e^x + 2xe^x - 4e^x = (2x-2)e^x$

Opgave 49:

a. 5,718
b. – 0,135
c. 20,086
d. 0,366
e. 9,852
f. – 26,229

Opgave 50:

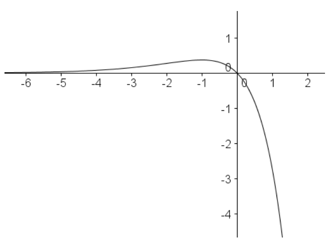
a. $f'(x) = -1 \cdot e^x + -x \cdot e^x = (-1-x)e^x = 0$

$-1 - x = 0 \quad \vee \quad e^x = 0$

$-x = 1$ k.n.

$x = -1$

$y = e^{-1} = \frac{1}{e}$



max $f(-1)=\frac{1}{e}$

b. $f'(0)=-1$

$y=-x+b$ door $(0,0)$

$0=b$

$k: \quad y=-x$

Opgave 51:

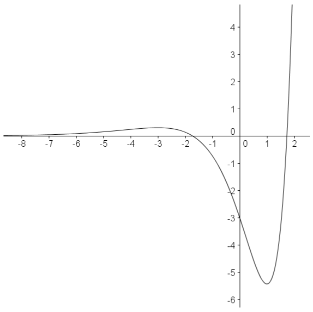
a. $(x^2-3)e^x=0$

$x^2-3=0 \quad \vee \quad e^x=0$

$x^2=3 \qquad$ k.n.

$x=\sqrt{3} \quad \vee \quad x=-\sqrt{3}$

b. $f'(x)=2x\cdot e^x+(x^2-3)\cdot e^x=(x^2+2x-3)e^x=0$



$x^2+2x-3=0 \quad \vee \quad e^x=0$

$(x+3)(x-1)=0 \qquad$ k.n.

$x=-3 \quad \vee \quad x=1$

$y=6e^{-3}=\frac{6}{e^3} \quad \vee \quad y=-2e$

max $f(-3)=\frac{6}{e^3}$

min $f(1)=-2e$

c. als $x\to-\infty$ dan $e^x\to 0$

e^x wint het van x^2-3 dus voor $x\to-\infty$ geldt $f(x)\to 0$

d. $p=\frac{6}{e^3} \quad \vee \quad -2e<p\le 0$

Opgave 52:

$$f(x)=\frac{2e^x}{e^x+1}$$

$$f'(x)=\frac{(e^x+1)\cdot 2e^x-2e^x\cdot e^x}{(e^x+1)^2}=\frac{2e^{2x}+2e^x-2e^{2x}}{(e^x+1)^2}=\frac{2e^x}{(e^x+1)^2}$$

$$f'(1)=\frac{2e}{(e+1)^2}$$

$$y_P=f(1)=\frac{2e}{e+1}$$

$$k: \quad y=\frac{2e}{(e+1)^2}\cdot x+b \text{ door } (1,\frac{2e}{e+1})$$

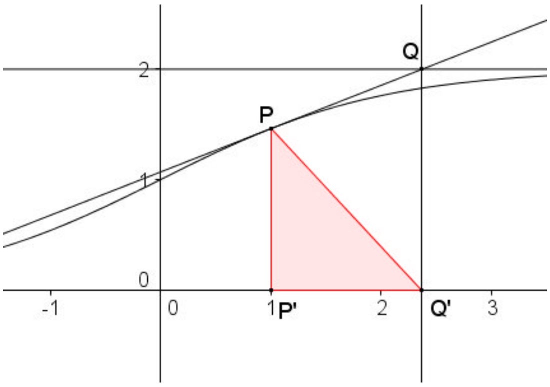
$$\frac{2e}{e+1}=\frac{2e}{(e+1)^2}+b$$

$$b=\frac{2e}{e+1}-\frac{2e}{(e+1)^2}=\frac{2e(e+1)}{(e+1)^2}-\frac{2e}{(e+1)^2}=\frac{2e^2+2e-2e}{(e+1)^2}=\frac{2e^2}{(e+1)^2}$$

$$k: \quad y=\frac{2e}{(e+1)^2}\cdot x+\frac{2e^2}{(e+1)^2}$$

k snijden met de lijn $y=2$ geeft:

$$\frac{2e}{(e+1)^2}\cdot x+\frac{2e^2}{(e+1)^2}=2$$
$$\frac{2e}{(e+1)^2}\cdot x=2-\frac{2e^2}{(e+1)^2}$$
$$\frac{2e}{(e+1)^2}\cdot x=\frac{2(e+1)^2}{(e+1)^2}-\frac{2e^2}{(e+1)^2}$$
$$\frac{2e}{(e+1)^2}\cdot x=\frac{2e^2+4e+2-2e^2}{(e+1)^2}$$
$$\frac{2e}{(e+1)^2}\cdot x=\frac{4e+2}{(e+1)^2}$$



$$x=\frac{4e+2}{(e+1)^2}\cdot\frac{(e+1)^2}{2e}=\frac{4e+2}{2e}=\frac{4e}{2e}+\frac{2}{2e}=2+\frac{1}{e}$$
$$x_{Q'}=2+\frac{1}{e}$$
$$P'Q'=x_{Q'}-x_{P'}=2+\frac{1}{e}-1=1+\frac{1}{e}=\frac{e}{e}+\frac{1}{e}=\frac{e+1}{e}$$
$$Opp(\Delta PP'Q')=\tfrac{1}{2}\cdot PP'\cdot P'Q'=\tfrac{1}{2}\cdot\frac{2e}{e+1}\cdot\frac{e+1}{e}=1$$

Opgave 53:

$f(x)=e^{ax+b}=e^u$ met $u=ax+b$ dus $u'=a$

$f'(x)=e^u\cdot u'=e^{ax+b}\cdot a=a\cdot e^{ax+b}$

Opgave 54:

a. $f(x)=e^{x^2+x}=e^u$ met $u=x^2+x$ dus $u'=2x+1$

$f'(x)=e^u\cdot u'=e^{x^2+x}\cdot(2x+1)=(2x+1)e^{x^2+x}$

b. $g(x)=x^2+2e^{3x}=x^2+2e^u$ met $u=3x$ dus $u'=3$

$g'(x)=2x+2e^u\cdot u'=2x+2e^{3x}\cdot 3=2x+6e^{3x}$

c. $h(x)=xe^{x^2}=x\cdot e^u$ met $u=x^2$ dus $u'=2x$

$h'(x)=1\cdot e^u+x\cdot e^u\cdot u'=e^{x^2}+xe^{x^2}\cdot 2x=e^{x^2}+2x^2e^{x^2}=(1+2x^2)e^{x^2}$

d. $j(x)=3x\cdot e^{2x-1}=3x\cdot e^u$ met $u=2x-1$ dus $u'=2$

$j'(x)=3\cdot e^u+3x\cdot e^u\cdot u'=3e^{2x-1}+3xe^{2x-1}\cdot 2=3e^{2x-1}+6xe^{2x-1}=(3+6x)e^{2x-1}$

e. $k(x)=\dfrac{2e^{-x-1}}{x^2}$

$T(x)=2e^{-x-1}=2e^u$ met $u=-x-1$ dus $u'=-1$

$T'(x)=2e^u\cdot u'=2e^{-x-1}\cdot -1=-2e^{-x-1}$

$$k'(x)=\frac{x^2\cdot -2e^{-x-1}-2e^{-x-1}\cdot 2x}{x^4}$$
$$=\frac{-2x^2e^{-x-1}-4xe^{-x-1}}{x^4}$$

$$= \frac{-2xe^{-x-1} - 4e^{-x-1}}{x^3}$$

$$= \frac{(-2x-4)e^{-x-1}}{x^3}$$

f. $l(x) = \dfrac{e^{2x}}{e^{2x}+1}$

$T(x) = e^{2x} = e^u$ met $u = 2x$ dus $u' = 2$

$T'(x) = e^u \cdot u' = e^{2x} \cdot 2 = 2e^{2x}$

$$l'(x) = \frac{(e^{2x}+1)\cdot 2e^{2x} - e^{2x}\cdot 2e^{2x}}{(e^{2x}+1)^2}$$

$$= \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x}+1)^2}$$

$$= \frac{2e^{2x}}{(e^{2x}+1)^2}$$

Opgave 55:

a. $f'(x) = \frac{1}{2}\cdot 2e^{2x} = e^{2x}$

$f'(-1) = e^{-2} = \dfrac{1}{e^2}$

$y_A = f(-1) = \frac{1}{2}e^{-2} = \dfrac{1}{2e^2}$

$k: \quad y = \dfrac{1}{e^2}\cdot x + b$ door $(-1, \dfrac{1}{2e^2})$

$\dfrac{1}{2e^2} = -\dfrac{1}{e^2} + b$

$b = \dfrac{1}{2e^2} + \dfrac{1}{e^2} = \dfrac{1}{2e^2} + \dfrac{2}{2e^2} = \dfrac{3}{2e^2}$

$k: \quad y = \dfrac{1}{e^2}\cdot x + \dfrac{3}{2e^2}$

$g(x) = \dfrac{1}{e^{x+3}} = e^{-x-3}$

$g'(x) = -e^{-x-3}$

$g'(-1) = -e^{-2} = -\dfrac{1}{e^2}$

$y_B = g(-1) = e^{-2} = \dfrac{1}{e^2}$

$l: \quad y = -\dfrac{1}{e^2}\cdot x + b$ door $(-1, \dfrac{1}{e^2})$

$\dfrac{1}{e^2} = \dfrac{1}{e^2} + b$

$b = 0$

$l: \quad y = -\dfrac{1}{e^2}\cdot x$

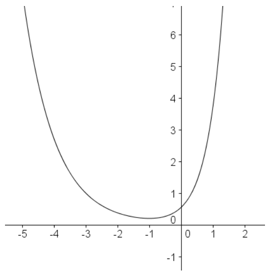
snijpunt van k en l:

$$\frac{1}{e^2} \cdot x + \frac{3}{2e^2} = -\frac{1}{e^2} \cdot x$$

$$\frac{2}{e^2} \cdot x = -\frac{3}{2e^2}$$

$$x = -\tfrac{3}{4}$$

b. $h(x) = f(x) + g(x)$



$$h'(x) = f'(x) + g'(x) = e^{2x} - e^{-x-3} = 0$$

$$e^{2x} = e^{-x-3}$$

$$2x = -x - 3$$

$$3x = -3$$

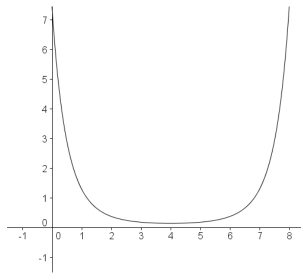
$$x = -1$$

$$\text{min } h(-1) = \tfrac{1}{2}e^{-2} + \frac{1}{e^2} = \frac{1}{2e^2} + \frac{1}{e^2} = \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}$$

$$B_h = [\frac{3}{2e^2}, \rightarrow\rangle$$

Opgave 56:

a. $f(x) = e^{\frac{1}{4}x^2 - 2x + 2}$



$$f'(x) = e^{\frac{1}{4}x^2 - 2x + 2} \cdot (\tfrac{1}{2}x - 2) = 0$$

$$e^{\frac{1}{4}x^2 - 2x + 2} = 0 \quad \vee \quad \tfrac{1}{2}x - 2 = 0$$

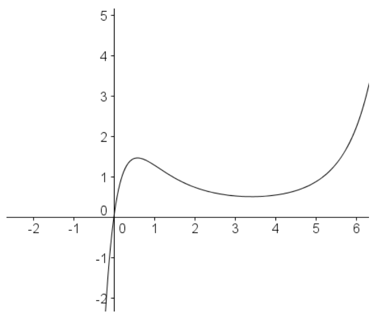
k.n. $\quad \tfrac{1}{2}x = 2$

$$x = 4$$

$$\text{min } f(4) = e^{-2} = \frac{1}{e^2}$$

$$B_f = \left[\frac{1}{e^2}, \rightarrow\right\rangle$$

b. $Opp(OPQR) = OP \cdot PQ = p \cdot e^{\frac{1}{4}p^2 - 2p + 2}$



$$Opp' = 1 \cdot e^{\frac{1}{4}p^2 - 2p + 2} + p \cdot e^{\frac{1}{4}p^2 - 2p + 2} \cdot (\tfrac{1}{2}p - 2) = 0$$

$$e^{\frac{1}{4}p^2 - 2p + 2} + \tfrac{1}{2}p^2 \cdot e^{\frac{1}{4}p^2 - 2p + 2} - 2p \cdot e^{\frac{1}{4}p^2 - 2p + 2} = 0$$

$$e^{\frac{1}{4}p^2 - 2p + 2} \cdot (1 + \tfrac{1}{2}p^2 - 2p) = 0$$

$$e^{\frac{1}{4}p^2 - 2p + 2} = 0 \quad \vee \quad \tfrac{1}{2}p^2 - 2p + 1 = 0$$

k.n. $\quad p^2 - 4p + 2 = 0$

$$p = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$$

de oppervlakte is minimaal voor $p = 2 - \sqrt{2}$

Opgave 57:

a. $f_1(x) = (x-1)^2 \cdot e^{2x}$

$$f_1'(x) = 2(x-1) \cdot e^{2x} + (x-1)^2 \cdot 2e^{2x}$$

$$= (2x-2) \cdot e^{2x} + 2(x-1)^2 \cdot e^{2x}$$

$= (2x^2 - 2x) \cdot e^{2x} = 0$

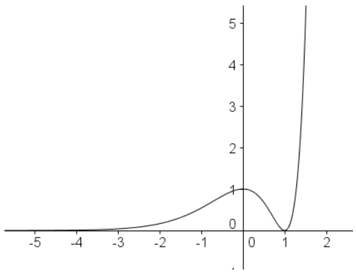
$2x^2 - 2x = 0 \quad \vee \quad e^{2x} = 0$

$2x(x-1) = 0 \qquad$ k.n.

$x = 0 \quad \vee \quad x = 1$

max $f_1(0) = 1$

min $f_1(1) = 0$



b. $f_a'(x) = 2(x-a) \cdot e^{2x} + (x-a)^2 \cdot 2e^{2x}$

$= (2x - 2a) \cdot e^{2x} + 2(x-a)^2 \cdot e^{2x}$

$= (2x^2 + 2x - 4ax + 2a^2 - 2a) \cdot e^{2x} = 0$

$2x^2 + 2x - 4ax + 2a^2 - 2a = 0 \quad \vee \quad e^{2x} = 0$

$2x^2 + (2-4a)x + 2a^2 - 2a = 0 \qquad$ k.n.

$x^2 + (1-2a)x + a^2 - a = 0$

$$x = \frac{-(1-2a) \pm \sqrt{(1-2a)^2 - 4(a^2 - a)}}{2} = \frac{-1 + 2a \pm \sqrt{1 - 4a + 4a^2 - 4a^2 + 4a}}{2}$$

$$= \frac{-1 + 2a \pm 1}{2}$$

$$x = \frac{-1 + 2a + 1}{2} = \frac{2a}{2} = a \quad \vee \quad x = \frac{-1 + 2a - 1}{2} = \frac{2a - 2}{2} = a - 1$$

$x_A = a - 1$ en $x_B = a$

c. $f(x_B) = f(a) = 0$ dus $y = 0$

d. $y = f(x_A) = f(a-1) = (a - 1 - a)^2 \cdot e^{2(a-1)} = e^{2(a-1)} = e^{2x}$

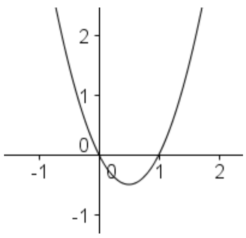
dus $y = e^{2x}$

e. $f_a'(0) = 2a^2 - 2a < 0$

$2a(a-1) = 0$

$a = 0 \quad \vee \quad a = 1$

$0 < a < 1$



9.4 De natuurlijke logaritme

Opgave 58:

a. $e^{{}^e\log 2 \cdot x} = (e^{{}^e\log 2})^x = 2^x$

b. $[2^x]' = [e^{{}^e\log 2 \cdot x}]' = [e^u]'$ met $u = {}^e\log 2 \cdot x$ dus $u' = {}^e\log 2$

$[e^u]' = e^u \cdot u' = e^{{}^e\log 2 \cdot x} \cdot {}^e\log 2 = 2^x \cdot {}^e\log 2$

Opgave 59:

a. $\ln e = \ln e^1 = 1$

b. $\ln e\sqrt{e} = \ln e^{1\frac{1}{2}} = 1\frac{1}{2}$

c. $\ln \frac{1}{e} = \ln e^{-1} = -1$

d. $1 = \ln e^0 = 0$

e. $3\ln e\sqrt[3]{e} = 3 \cdot \ln e^{1\frac{1}{3}} = 3 \cdot 1\frac{1}{3} = 4$

f. $\ln^2 e^3 = (\ln e^3)^2 = 3^2 = 9$

g. $\ln^3 e^2 = (\ln e^2)^3 = 2^3 = 8$

h. $e^{\ln 7} + e^{2\ln 7} = e^{\ln 7} + e^{\ln 7^2} = 7 + 7^2 = 7 + 49 = 56$

i. $e^{\frac{1}{2}\ln 5} = e^{\ln 5^{\frac{1}{2}}} = 5^{\frac{1}{2}} = \sqrt{5}$

j. $e^{\ln 10} \cdot e^{\ln 3} = 10 \cdot 3 = 30$

Opgave 60:

a. $e^{3x} = 12$

$3x = \ln 12$

$x = \frac{1}{3}\ln 12$

b. $5e^{2x} = 60$

$e^{2x} = 12$

$2x = \ln 12$

$x = \frac{1}{2}\ln 12$

c. $6 + e^{0{,}5x} = 10$

$e^{0{,}5x} = 4$

$0{,}5x = \ln 4$

$x = 2\ln 4$

d. $\dfrac{3}{e^{2x}} = 10$

$e^{2x} = \frac{3}{10}$

$2x = \ln \frac{3}{10}$

$x = \frac{1}{2}\ln \frac{3}{10}$

Opgave 61:

a. $2\ln 3 + \ln 4 = \ln 3^2 + \ln 4 = \ln 9 + \ln 4 = \ln 36$

b. $\ln 20 - 3\ln 2 = \ln 20 - \ln 2^3 = \ln 20 - \ln 8 = \ln \frac{20}{8} = \ln 2\frac{1}{2}$

c. $4 + \ln 3 = \ln e^4 + \ln 3 = \ln 3e^4$

d. $1 + \ln 10 = \ln e + \ln 10 = \ln 10e$

e. $\frac{1}{2}+2\ln 6=\ln\sqrt{e}+\ln 6^2=\ln\sqrt{e}+\ln 36=\ln 36\sqrt{e}$

f. $e+\ln 2=\ln e^e+\ln 2=\ln 2e^e$

Opgave 62:

a. $\ln x=-1$
$x=e^{-1}=\frac{1}{e}$

b. $4\ln x=2$
$\ln x=\frac{1}{2}$
$x=e^{\frac{1}{2}}=\sqrt{e}$

c. $\ln 3x=3$
$3x=e^3$
$x=\frac{1}{3}e^3$

d. $\ln(-x+2)=-2$
$-x+2=e^{-2}=\frac{1}{e^2}$
$-x=\frac{1}{e^2}-2$
$x=2-\frac{1}{e^2}$

e. $\ln^2 x=\frac{1}{4}$
$\ln x=\frac{1}{2} \quad\vee\quad \ln x=-\frac{1}{2}$
$x=e^{\frac{1}{2}}=\sqrt{e} \quad\vee\quad x=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$

f. $\ln x=1+\ln 5$
$\ln x=\ln e+\ln 5$
$\ln x=\ln 5e$
$x=5e$

Opgave 63:

a. $4e^{1-3x}=20$
$e^{1-3x}=5$
$1-3x=\ln 5$
$-3x=-1+\ln 5$
$x=\frac{1}{3}-\frac{1}{3}\ln 5=-0{,}203$

b. $e^{x^2}=100$
$x^2=\ln 100$
$x=\sqrt{\ln 100}=2{,}146 \quad\vee\quad x=-\sqrt{\ln 100}=-2{,}146$

Opgave 64:

a. $3x\ln x=2\ln x$
$3x\ln x-2\ln x=0$
$\ln x\cdot(3x-2)=0$
$\ln x=0 \quad\vee\quad 3x=2$
$x=e^0=1 \quad\vee\quad x=\frac{2}{3}$

b. $\ln^2 x-\ln x=0$
$\ln x\cdot(\ln x-1)=0$

$\ln x = 0 \quad \vee \quad \ln x = 1$

$x = e^0 = 1 \quad \vee \quad x = e$

c. $x^2 \ln(x+1) = 4\ln(x+1)$

$x^2 \ln(x+1) - 4\ln(x+1) = 0$

$\ln(x+1) \cdot (x^2 - 4) = 0$

$\ln(x+1) = 0 \quad \vee \quad x^2 - 4 = 0$

$x + 1 = e^0 = 1 \quad \vee \quad x^2 = 4$

$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -2$ (vervalt)

$x = 0 \quad \vee \quad x = 2$

d. $\ln^2 x - 2\ln x - 3 = 0$

neem $p = \ln x$ dan $p^2 - 2p - 3 = 0$

$(p-3)(p+1) = 0$

$p = 3 \quad \vee \quad p = -1$

$\ln x = 3 \quad \vee \quad \ln x = -1$

$x = e^3 \quad \vee \quad x = e^{-1} = \frac{1}{e}$

e. $\ln(x+3) - \ln(x-1) = \ln 2$

$\ln \frac{x+3}{x-1} = \ln 2$

$\frac{x+3}{x-1} = 2$

$2(x-1) = x + 3$

$2x - 2 = x + 3$

$x = 5$

f. $2\ln x = \ln 2 + \ln(x+4)$

$\ln x^2 = \ln 2(x+4)$

$x^2 = 2(x+4)$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \quad \vee \quad x = -2$ (vervalt)

$x = 4$

Opgave 65:

a. $f(x) = 3^{4x-3} = 3^u$ met $u = 4x - 3$ dus $u' = 4$

$f'(x) = 3^u \cdot \ln 3 \cdot u' = 3^{4x-3} \cdot \ln 3 \cdot 4 = 4\ln 3 \cdot 3^{4x-3}$

b. $g(x) = (2x-1) \cdot 2^x$

$g'(x) = 2 \cdot 2^x + (2x-1) \cdot 2^x \cdot \ln 2$

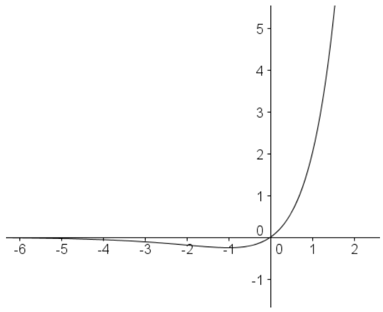
c. $h(x) = \frac{2^x + 1}{2^x - 1}$

$$h'(x) = \frac{(2^x - 1) \cdot 2^x \cdot \ln 2 - (2^x + 1) \cdot 2^x \cdot \ln 2}{(2^x - 1)^2} = \frac{2^{2x} \cdot \ln 2 - 2^x \cdot \ln 2 - 2^{2x} \cdot \ln 2 - 2^x \cdot \ln 2}{(2^x - 1)^2}$$

$$= \frac{-2 \cdot \ln 2 \cdot 2^x}{(2^x - 1)^2}$$

Opgave 66:

a. $f(x) = 2^{2x} - 2^x$



$f'(x) = 2^{2x} \cdot \ln 2 \cdot 2 - 2^x \cdot \ln 2 = (2 \cdot 2^{2x} - 2^x) \cdot \ln 2 = 0$

$2 \cdot 2^{2x} - 2^x = 0$

$2^{2x+1} = 2^x$

$2x + 1 = x$

$x = -1$

$y = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

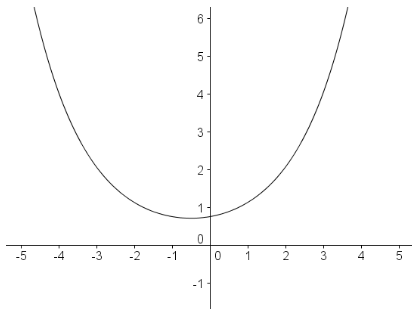
$B_g = [-\frac{1}{4}, \rightarrow\rangle$

b. $f'(0) = \ln 2$

dus $0 < a < \ln 2 \quad \vee \quad a > \ln 2$

Opgave 67:

a. $f(x) = 2^{x-1} + 2^{-x-2}$



$f'(x) = 2^{x-1} \cdot \ln 2 + 2^{-x-2} \cdot \ln 2 \cdot -1 = (2^{x-1} - 2^{-x-2}) \cdot \ln 2 = 0$

$2^{x-1} - 2^{-x-2} = 0$

$2^{x-1} = 2^{-x-2}$

$x - 1 = -x - 2$

$2x = -1$

$x = -\frac{1}{2}$

$y = 2^{-1\frac{1}{2}} + 2^{-1\frac{1}{2}} = 2 \cdot 2^{-1\frac{1}{2}} = 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$

$B_f = [\frac{1}{2}\sqrt{2}, \rightarrow\rangle$

b. $f'(x) = (2^{x-1} - 2^{-x-2}) \cdot \ln 2 = -\frac{1}{4}\ln 2$

$2^{x-1} - 2^{-x-2} = -\frac{1}{4}$

$2^{-1} \cdot 2^x - 2^{-2} \cdot 2^x + \frac{1}{4} = 0$

$\frac{1}{2} \cdot 2^x - \frac{1}{4} \cdot \frac{1}{2^x} + \frac{1}{4} = 0$

neem $p = 2^x$ dan $\frac{1}{2}p - \frac{1}{4} \cdot \frac{1}{p} + \frac{1}{4} = 0$

$2p - \frac{1}{p} + 1 = 0$

$2p^2 - 1 + p = 0$

$p = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$

$p = -1 \quad \vee \quad p = \frac{1}{2}$

$2^x = -1 \quad \vee \quad 2^x = \frac{1}{2}$

k.n. $\qquad 2^x = 2^{-1}$

$x = -1$ dan $y = \frac{3}{4}$ dus $(-1, \frac{3}{4})$

c. $f'(x) = (2^{x-1} - 2^{-x-2}) \cdot \ln 2 = -3$

$y_1 = (2^{x-1} - 2^{-x-2}) \cdot \ln 2$ en $y_2 = -3$

intersect geeft $x = -4{,}1233$

$y = 4{,}3855$

$b = y + 3x = 4{,}3855 + 3 \cdot -4{,}1233 = -7{,}984$

Opgave 68:

a. $[x]' = [e^{\ln x}]' = [e^u]'$ met $u = \ln x$ en $u' = [\ln x]'$

$[e^u]' = e^u \cdot u' = e^{\ln x} \cdot [\ln x]' = x \cdot [\ln x]'$

ook geldt: $[x]' = 1$

b. $x \cdot [\ln x]' = 1$

dus $[\ln x]' = \dfrac{1}{x}$

c. $g(x) = \dfrac{{}^e\log x}{{}^e\log 2} = \dfrac{\ln x}{\ln 2} = \dfrac{1}{\ln 2} \cdot \ln x$

$g'(x) = \dfrac{1}{\ln 2} \cdot \dfrac{1}{x} = \dfrac{1}{x \ln 2}$

Opgave 69:

a. $[\ln 6x]' = [\ln 6 + \ln x]' = [\ln 6]' + [\ln x]' = 0 + \dfrac{1}{x} = \dfrac{1}{x}$

b. $f'(x) = \dfrac{1}{x}$

$g'(x) = \dfrac{1}{x}$

$h'(x) = \dfrac{1}{x \ln 2}$

Opgave 70:

a. $[\ln x^6]' = [6 \cdot \ln x]' = 6 \cdot [\ln x]' = 6 \cdot \dfrac{1}{x} = \dfrac{6}{x}$

b. $f'(x) = \dfrac{2}{x}$

$g'(x) = \dfrac{-3}{x}$

$h'(x) = \dfrac{-1}{x}$

Opgave 71:

a. $f(x) = \dfrac{1 - \ln x}{x}$

$$f'(x) = \frac{x \cdot \frac{-1}{x} - (1 - \ln x) \cdot 1}{x^2} = \frac{-1 - 1 + \ln x}{x^2} = \frac{-2 + \ln x}{x^2}$$

b. $g(x)=x\ln x$

$$g'(x)=1\cdot\ln x+x\cdot\frac{1}{x}=\ln x+1$$

c. $f(x)={}^2\log(4x-1)$

$$f'(x)=\frac{1}{4x-1}\cdot 4\cdot\tfrac{1}{\ln 2}=\frac{4}{(4x-1)\cdot\ln 2}$$

d. $f(x)=\dfrac{\ln 3x}{x}$

$$f'(x)=\frac{x\cdot\frac{1}{x}-\ln 3x\cdot 1}{x^2}=\frac{1-\ln 3x}{x^2}$$

e. $f(x)=x\cdot\ln x^3$

$$f'(x)=1\cdot\ln x^3+x\cdot\frac{3}{x}=\ln x^3+3$$

f. $f(x)={}^3\log x^2$

$$f'(x)=\frac{1}{x^2}\cdot 2x\cdot\tfrac{1}{\ln 3}=\frac{2}{x\ln 3}$$

Opgave 72:

a. $f(x)=\ln(x^2+2x)$

$$f'(x)=\frac{1}{x^2+2x}\cdot(2x+1)=\frac{2x+1}{x^2+2x}$$

b. $g(x)=\ln 2^x=x\cdot\ln 2$

$g'(x)=\ln 2$

c. $h(x)={}^2\log(x^2+1)$

$$h'(x)=\frac{1}{x^2+1}\cdot 2x\cdot\frac{1}{\ln 2}=\frac{2x}{(x^2+1)\cdot\ln 2}$$

d. $j(x)=\log 4x^2$

$$j'(x)=\frac{1}{4x^2}\cdot 8x\cdot\frac{1}{\ln 10}=\frac{2}{x\ln 10}$$

Opgave 73:

a. $f(x)=x\cdot\ln^2 x$

$$f'(x)=1\cdot\ln^2 x+x\cdot 2\ln x\cdot\frac{1}{x}=\ln^2 x+2\ln x$$

b. $g(x)=x^2\cdot{}^3\log 4x$

$$g'(x)=2x\cdot{}^3\log 4x+x^2\cdot\frac{1}{4x}\cdot 4\cdot\frac{1}{\ln 3}=2x\cdot{}^3\log 4x+\frac{x}{\ln 3}$$

c. $h(x)=\log^2(4x)$

$$h'(x)=2\cdot\log 4x\cdot\frac{1}{4x}\cdot 4\cdot\frac{1}{\ln 10}=\frac{2\cdot\log 4x}{x\ln 10}$$

d. $j(x) = \ln^2(4x^2+1)$

$j'(x) = 2\ln(4x^2+1) \cdot \frac{1}{4x^2+1} \cdot 8x = \frac{16x\ln(4x^2+1)}{4x^2+1}$

Opgave 74:

a. $e^{n \cdot \ln x} = e^{\ln x^n} = x^n$

b. $y = e^{n \cdot \ln x} = e^u$ met $u = n \cdot \ln x$ dus $u' = n \cdot \frac{1}{x}$

c. $y' = [e^{n \cdot \ln x}]' = e^u \cdot u' = e^{n \cdot \ln x} \cdot n \cdot \frac{1}{x} = e^{n \cdot \ln x} \cdot \frac{n}{x}$

d. $[x^n]' = [(e^{\ln x})^n]' = [e^{n \cdot \ln x}]' = e^{n \cdot \ln x} \cdot \frac{n}{x} = (e^{\ln x})^n \cdot \frac{n}{x} = x^n \cdot \frac{n}{x} = n \cdot x^{n-1}$

er is geen beperking voor de waarde van n

Opgave 75:

a. $f(x) = \frac{10\ln x}{x} = 0$

$10\ln x = 0$

$\ln x = 0$

$x = e^0 = 1$

$f'(x) = \frac{x \cdot \frac{10}{x} - 10\ln x}{x^2} = \frac{10 - 10\ln x}{x^2}$

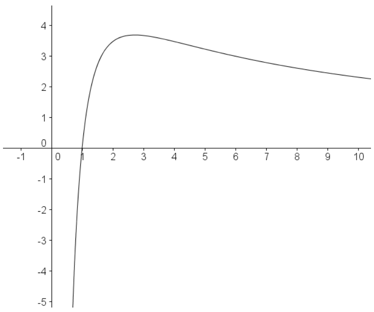
$f'(1) = 10$

$y = 10x + b$ door $(1,0)$

$0 = 10 + b$

$b = -10$

$y = 10x - 10$



b. $f'(x) = \frac{10 - 10\ln x}{x^2} = 0$

$10 - 10\ln x = 0$

$-10\ln x = -10$

$\ln x = 1$

$x = e$

max $f(e) = \frac{10}{e}$

c. stel $x_B = p$ dan $x_C = 2p$

$y_B = y_C$

$f(p) = f(2p)$

$\frac{10\ln p}{p} = \frac{10\ln 2p}{2p}$

$\frac{20\ln p}{2p} = \frac{10\ln 2p}{2p}$

$20\ln p = 10\ln 2p$

$2\ln p = \ln 2p$

$\ln p^2 = \ln 2p$

$p^2 = 2p$

$p^2 - 2p = 0$

$p(p-2) = 0$

$p = 0 \quad \vee \quad p = 2$

vervalt $\quad q = f(2) = \dfrac{10\ln 2}{2} = 5\ln 2$

Opgave 76:

a. $f(x) = \dfrac{x}{\ln x}$

$y_A = f(\tfrac{1}{e}) = -\tfrac{1}{e}$

$f'(x) = \dfrac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{\ln^2 x} = \dfrac{\ln x - 1}{\ln^2 x}$

$f'(\tfrac{1}{e}) = \dfrac{-1-1}{1} = -2$

$y = -2x + b$ door $(\tfrac{1}{e}, -\tfrac{1}{e})$

$-\tfrac{1}{e} = -\tfrac{2}{e} + b$

$b = \tfrac{1}{e}$

$y = -2x + \tfrac{1}{e}$

b. $f'(x) = \dfrac{\ln x - 1}{\ln^2 x} = -6$

$\ln x - 1 = -6\ln^2 x$

$6\ln^2 x + \ln x - 1 = 0$

neem $p = \ln x$ dan $6p^2 + p - 1 = 0$

$p = \dfrac{-1 \pm \sqrt{25}}{12}$

$p = \dfrac{-1-5}{12} = -\tfrac{1}{2} \quad \vee \quad p = \dfrac{-1+5}{12} = \dfrac{1}{3}$

$\ln x = -\tfrac{1}{2} \quad \vee \quad \ln x = \tfrac{1}{3}$

$x = e^{-\frac{1}{2}} = \tfrac{1}{\sqrt{e}} \quad \vee \quad x = e^{\frac{1}{3}} = \sqrt[3]{e}$

$y = \tfrac{-2}{\sqrt{e}} \qquad\qquad y = 3 \cdot \sqrt[3]{e}$

$(\tfrac{1}{\sqrt{e}}, \tfrac{-2}{\sqrt{e}})$ en $(\sqrt[3]{e}, 3 \cdot \sqrt[3]{e})$

Opgave 77:

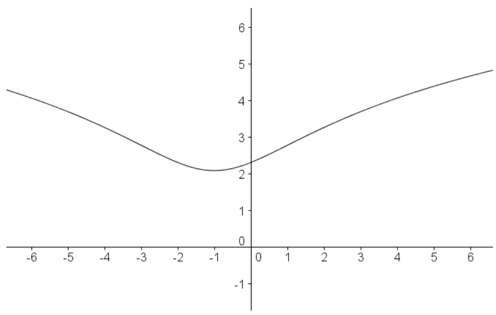
a. $f(x) = \ln(2x^2 + 4x + 10)$

$f'(x) = \dfrac{1}{2x^2 + 4x + 10} \cdot (4x + 4) = \dfrac{4x+4}{2x^2 + 4x + 10} = 0$

$4x + 4 = 0$

$4x = -4$

$x = -1$



$y=\ln 8$

$B_f=[\ln 8,\rightarrow\rangle$

b. $f'(x)=\dfrac{4x+4}{2x^2+4x+10}=\frac{2}{5}$

$2(2x^2+4x+10)=5(4x+4)$

$4x^2+8x+20=20x+20$

$4x^2-12x=0$

$4x(x-3)=0$

$x=0 \quad \vee \quad x=3$

$y=\ln 10 \qquad y=\ln 40$

$(0,\ln 10) \qquad (3,\ln 40)$

c. $f'(x)=\dfrac{4x+4}{2x^2+4x+10}=1$

$2x^2+4x+10=4x+4$

$2x^2=-6$

kan niet, dus geen oplossingen

Opgave 78:

a. $\ln 2x=\ln\dfrac{4}{x}$

$2x=\dfrac{4}{x}$

$2x^2=4$

$x^2=2$

$x=\sqrt{2} \quad \vee \quad x=-\sqrt{2}$ (vervalt)

$y=\ln 2\sqrt{2}$

$(\sqrt{2},\ln 2\sqrt{2})$

b. $\ln\dfrac{4}{x}=0$

$\dfrac{4}{x}=e^0=1$

$x=4$

$g(x)=\ln\dfrac{4}{x}=\ln 4-\ln x$

$g'(x)=-\dfrac{1}{x}$

$g'(4)=-\frac{1}{4}$

$y=-\frac{1}{4}x+b$ door $(4,0)$

$0=-1+b$

$b=1$

$y=-\frac{1}{4}x+1$

c. $f(x)-g(x)=2 \quad \vee \quad g(x)-f(x)=2$

$$\ln 2x - \ln\frac{4}{x} = 2 \quad \vee \quad \ln\frac{4}{x} - \ln 2x = 2$$

$$\ln\frac{2x}{\frac{4}{x}} = 2 \quad \vee \quad \ln\frac{\frac{4}{x}}{2x} = 2$$

$$\ln \tfrac{1}{2}x^2 = 2 \quad \vee \quad \ln\frac{2}{x^2} = 2$$

$$\tfrac{1}{2}x^2 = e^2 \quad \vee \quad \frac{2}{x^2} = e^2$$

$$x^2 = 2e^2 \quad \vee \quad x^2 = \tfrac{2}{e^2}$$

$x = e\sqrt{2} \quad \vee \quad x = -e\sqrt{2}$ (vervalt) $\quad \vee \quad x = \tfrac{1}{e}\sqrt{2} \quad \vee \quad x = -\tfrac{1}{e}\sqrt{2}$ (vervalt)

dus $x = e\sqrt{2} \quad \vee \quad x = \tfrac{1}{e}\sqrt{2}$

d. $y_B = \ln 2x$ en $y_C = \ln\frac{4}{x}$

$y_M = \frac{y_B + y_C}{2} = \tfrac{1}{2}(y_B + y_C) = \tfrac{1}{2}(\ln 2x + \ln\frac{4}{x}) = \tfrac{1}{2}(\ln 8) = \tfrac{1}{2}\ln 8$ dus onafhankelijk van p

9.5 Diagnostische toets

Opgave 1:

a. $^{3}\log 5 + 2\cdot{}^{3}\log 2 = {}^{3}\log 5 + {}^{3}\log 2^2 = {}^{3}\log 5 + {}^{3}\log 4 = {}^{3}\log 20$

b. $3 - {}^{2}\log 5 = {}^{2}\log 8 - {}^{2}\log 5 = {}^{2}\log \frac{8}{5}$

c. $^{2}\log 8000 + 3\cdot{}^{2}\log \frac{1}{5} = {}^{2}\log 8000 + {}^{2}\log(\frac{1}{5})^3 = {}^{2}\log 8000 + {}^{2}\log \frac{1}{125} = {}^{2}\log 64 = 6$

Opgave 2:

a. $2\cdot{}^{2}\log(x-1) = 1 + {}^{2}\log 18$

$^{2}\log(x-1)^2 = {}^{2}\log 2 + {}^{2}\log 18$

$^{2}\log(x-1)^2 = {}^{2}\log 36$

$(x-1)^2 = 36$

$x - 1 = 6 \quad \vee \quad x - 1 = -6$

$x = 7 \quad \vee \quad x = -5$ (vervalt)

b. $^{2}\log x = 3 - {}^{2}\log(x+2)$

$^{2}\log x = {}^{2}\log 8 - {}^{2}\log(x+2)$

$$^{2}\log x = {}^{2}\log \frac{8}{x+2}$$

$$x = \frac{8}{x+2}$$

$x(x+2) = 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ (vervalt) $\quad \vee \quad x = 2$

Opgave 3:

a. $^{2}\log x - {}^{\frac{1}{2}}\log(x-1) = 3$

$$^{2}\log x - \frac{^{2}\log(x-1)}{^{2}\log \frac{1}{2}} = 3$$

$^{2}\log x + {}^{2}\log(x-1) = 3$

$^{2}\log x(x-1) = {}^{2}\log 8$

$x(x-1) = 8$

$x^2 - x - 8 = 0$

$$x = \frac{1 \pm \sqrt{33}}{2}$$

$x = \frac{1}{2} + \frac{1}{2}\sqrt{33} \quad \vee \quad x = \frac{1}{2} - \frac{1}{2}\sqrt{33}$ (vervalt)

b. $\log^2 x - 5\cdot\log x = 6$

neem $p = \log x$ dan $p^2 - 5p = 6$

$p^2 - 5p - 6 = 0$

$(p-6)(p+1) = 0$

$p = 6 \quad \vee \quad p = -1$

$\log x = 6 \quad \vee \quad \log x = -1$

$x = 10^6 \quad \vee \quad x = 10^{-1} = \frac{1}{10}$

Opgave 4:

a. $3^x + 6 \cdot (\frac{1}{3})^x = 5$

$3^x + 6 \cdot \dfrac{1}{3^x} - 5 = 0$

neem $p = 3^x$ dan $p + 6 \cdot \dfrac{1}{p} - 5 = 0$

$p^2 + 6 - 5p = 0$

$(p-2)(p-3) = 0$

$p = 2 \quad \vee \quad p = 3$

$3^x = 2 \quad \vee \quad 3^x = 3$

$x = {}^3\log 2 \quad \vee \quad x = 1$

b. $9^x = 3^x + 12$

$(3^x)^2 - 3^x - 12 = 0$

neem $p = 3^x$ dan $p^2 - p - 12 = 0$

$(p-4)(p+3) = 0$

$p = 4 \quad \vee \quad p = -3$

$3^x = 4 \quad \vee \quad 3^x = -3$

$x = {}^3\log 4$ k.n.

dus $x = {}^3\log 4$

c. $9^x = 3^{x+1} + 4$

$(3^x)^2 = 3 \cdot 3^x + 4$

neem $p = 3^x$ dan $p^2 = 3p + 4$

$p^2 - 3p - 4 = 0$

$(p-4)(p+1) = 0$

$p = 4 \quad \vee \quad p = -1$

$3^x = 4 \quad \vee \quad 3^x = -1$

$x = {}^3\log 4$ k.n.

dus $x = {}^3\log 4$

d. $3^{x+2} + 3^{2x+1} = 12$

$3^2 \cdot 3^x + 3 \cdot 3^{2x} = 12$

$9 \cdot 3^x + 3 \cdot (3^x)^2 = 12$

neem $p = 3^x$ dan $9p + 3p^2 = 12$

$3p^2 + 9p - 12 = 0$

$p^2 + 3p - 4 = 0$

$(p+4)(p-1) = 0$

$p = -4 \quad \vee \quad p = 1$

$3^x = -4 \quad \vee \quad 3^x = 1$

k.n. $x = 0$

Opgave 5:

a. $y = 3^x \xrightarrow{V_{x-as,\frac{1}{3}}} y = \frac{1}{3} \cdot 3^x = 3^{-1} \cdot 3^x = 3^{x-1}$

dus $T(1,0)$

b. $y = {}^3\log x \xrightarrow{T(0,-2)} y = -2 + {}^3\log x = {}^3\log \frac{1}{9} + {}^3\log x = {}^3\log \frac{1}{9} x$

dus $V_{y-as,9}$

Opgave 6:

a. $f(x) - g(x) = 2 \quad \vee \quad g(x) - f(x) = 2$

$3^{x-1} - 4 - (2 - 3^x) = 2 \quad \vee \quad 2 - 3^x - (3^{x-1} - 4) = 2$

$3^{x-1} - 4 - 2 + 3^x = 2 \quad \vee \quad 2 - 3^x - 3^{x-1} + 4 = 2$

$3^{-1} \cdot 3^x + 3^x = 8 \quad \vee \quad -3^x - 3^{x-1} = -4$

$\frac{1}{3} \cdot 3^x + 3^x = 8 \quad \vee \quad 3^x + 3^{x-1} = 4$

$\frac{4}{3} \cdot 3^x = 8 \quad \vee \quad 3^x + 3^{-1} \cdot 3^x = 4$

$3^x = 6 \quad \vee \quad 3^x + \frac{1}{3} \cdot 3^x = 4$

$x = {}^3\log 6 \quad \vee \quad \frac{4}{3} \cdot 3^x = 4$

$3^x = 3$

$x = 1$

dus $p = {}^3\log 6 \quad \vee \quad p = 1$

b. $f(x) = g(x+1) \quad \vee \quad g(x) = f(x+1)$

$3^{x-1} - 4 = 2 - 3^{x+1} \quad \vee \quad 2 - 3^x = 3^x - 4$

$3^{x-1} + 3^{x+1} = 6 \quad \vee \quad -2 \cdot 3^x = -6$

$3^{-1} \cdot 3^x + 3 \cdot 3^x = 6 \quad \vee \quad 3^x = 3$

$\frac{1}{3} \cdot 3^x + 3 \cdot 3^x = 6 \quad \vee \quad x = 1$

$3\frac{1}{3} \cdot 3^x = 6 \qquad y = 2 - 3 = -1$

$3^x = 1\frac{4}{5}$

$x = {}^3\log 1\frac{4}{5}$

$y = 3^{{}^3\log 1\frac{4}{5} - 1} - 4 = 3^{{}^3\log 1\frac{4}{5}} \cdot 3^{-1} - 4 = 1\frac{4}{5} \cdot \frac{1}{3} - 4 = -3\frac{2}{5}$

$q = -3\frac{2}{5} \quad \vee \quad q = -1$

Opgave 7:

a. $x_B = p$ dan $x_C = 3p$

$g(x_B) = f(x_C)$

${}^2\log 4p = {}^2\log(3p + 3)$

$4p = 3p + 3$

$p = 3$

$q = {}^2\log 12$

b. $f(p) = 2 \cdot g(p)$

${}^2\log(p + 3) = 2 \cdot {}^2\log 4p$

${}^2\log(p + 3) = {}^2\log(4p)^2$

$$p+3=(4p)^2$$
$$p+3=16p^2$$
$$16p^2-p-3=0$$
$$p=\frac{1\pm\sqrt{193}}{32}$$
$$p=\frac{1+\sqrt{193}}{32}=0{,}47 \quad \vee \quad p=\frac{1-\sqrt{193}}{32}=-0{,}40 \text{ (vervalt)}$$

Opgave 8:

a. $\dfrac{3e^3-e^3}{e^2}=\dfrac{2e^3}{e^2}=2e$

b. $(e^{3x}-5)^2=e^{6x}-10e^{3x}+25$

Opgave 9:

a. $3xe^x-e^x=0$

$e^x(3x-1)=0$

$e^x=0 \quad \vee \quad 3x=1$

k.n. $\quad x=\frac{1}{3}$

b. $e^{2x-1}-\sqrt[3]{e^2}=0$

$e^{2x-1}=\sqrt[3]{e^2}$

$e^{2x-1}=e^{\frac{2}{3}}$

$2x-1=\frac{2}{3}$

$2x=\frac{5}{3}$

$x=\frac{5}{6}$

c. $e^{4x}-e^{x+1}=0$

$e^{4x}=e^{x+1}$

$4x=x+1$

$3x=1$

$x=\frac{1}{3}$

d. $e^{2x}+2e^x=3$

$(e^x)^2+2e^x-3=0$

neem $p=e^x$ dan $p^2+2p-3=0$

$(p+3)(p-1)=0$

$p=-3 \quad \vee \quad p=1$

$e^x=-3 \quad \vee \quad e^x=1$

k.n. $\quad x=0$

Opgave 10:

a. $f(x)=2e^x-3x^2$

$f'(x)=2e^x-6x$

b. $f(x) = \frac{x^2+1}{e^x}$

$f'(x) = \frac{e^x \cdot 2x - (x^2+1)\cdot e^x}{(e^x)^2} = \frac{2x-(x^2+1)}{e^x} = \frac{-x^2+2x-1}{e^x}$

c. $f(x) = (x^2+1)e^x$

$f'(x) = 2x \cdot e^x + (x^2+1)e^x = (x^2+2x+1)e^x$

d. $f(x) = \frac{e^x}{x^2+1}$

$f'(x) = \frac{(x^2+1)e^x - e^x \cdot 2x}{(x^2+1)^2} = \frac{x^2e^x - 2xe^x + e^x}{(x^2+1)^2} = \frac{(x^2-2x+1)e^x}{(x^2+1)^2}$

e. $f(x) = x^2 e^{2x-1}$

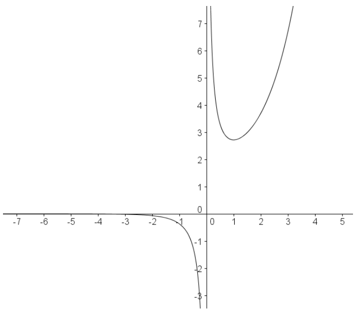
$f'(x) = 2xe^{2x-1} + x^2e^{2x-1} \cdot 2 = (2x^2+2x)e^{2x-1}$

f. $f(x) = e^{x^2+9}$

$f'(x) = e^{x^2+9} \cdot 2x = 2xe^{x^2+9}$

Opgave 11:

a. $f(x) = \frac{e^x}{x}$



$f'(x) = \frac{x \cdot e^x - e^x \cdot 1}{x^2} = \frac{(x-1)e^x}{x^2} = 0$

$(x-1)e^x = 0$

$x - 1 = 0 \quad \vee \quad e^x = 0$

$x = 1$ k.n.

$y = e$

min $f(1) = e$

b. $x_A = 2 \quad y_A = \frac{1}{2}e^2$

$f'(2) = \frac{1}{4}e^2$

$y = \frac{1}{4}e^2 \cdot x + b$ door $(2, \frac{1}{2}e^2)$

$\frac{1}{2}e^2 = \frac{1}{2}e^2 + b$

$b = 0$

$l: \quad y = \frac{1}{2}e^2 \cdot x$

Opgave 12:

a. $\ln(e^3 \cdot \sqrt{e}) = \ln e^{3\frac{1}{2}} = 3\frac{1}{2}$

b. $\ln \frac{1}{e^2} = \ln e^{-2} = -2$

Opgave 13:

a. $4 + \ln 3 = \ln e^4 + \ln 3 = \ln 3e^4$

b. $\ln 10 - 4\ln 2 = \ln 10 - \ln 2^4 = \ln 10 - \ln 16 = \ln \frac{10}{16} = \ln \frac{5}{8}$

Opgave 14:

a. $2\ln 5x = 16$

$\ln 5x = 8$

$5x = e^8$

$x = \frac{1}{5}e^8$

b. $\ln^2 5x = 16$

$\ln 5x = 4 \quad \vee \quad \ln 5x = -4$

$5x = e^4 \quad \vee \quad 5x = e^{-4} = \frac{1}{e^4}$

$x = \frac{1}{5}e^4 \quad \vee \quad x = \frac{1}{5e^4}$

c. $2\ln^2 x - \ln x = 0$

$\ln x \cdot (2\ln x - 1) = 0$

$\ln x = 0 \quad \vee \quad 2\ln x = 1$

$x = 1 \quad \vee \quad \ln x = \frac{1}{2}$

$x = 1 \quad \vee \quad x = e^{\frac{1}{2}} = \sqrt{e}$

d. $\ln(9x+1) - \ln(x+2) = \ln 4$

$\ln \frac{9x+1}{x+2} = \ln 4$

$\frac{9x+1}{x+2} = 4$

$9x + 1 = 4(x+2)$

$9x + 1 = 4x + 8$

$5x = 7$

$x = 1\frac{2}{5}$

Opgave 15:

a. $f(x) = 2^{3x-4}$

$f'(x) = 2^{3x-4} \cdot \ln 2 \cdot 3 = 3\ln 2 \cdot 2^{3x-4}$

b. $f(x) = x \cdot 3^x$

$f'(x) = 1 \cdot 3^x + x \cdot 3^x \cdot \ln 3 = (1 + x\ln 3) \cdot 3^x$

c. $f(x) = \ln(x \cdot \sqrt[3]{x}) = \ln x^{\frac{4}{3}} = \frac{4}{3}\ln x$

$f'(x) = \frac{4}{3} \cdot \frac{1}{x} = \frac{4}{3x}$

d. $f(x) = {}^2\log 4x$

$f'(x) = \frac{1}{4x} \cdot 4 \cdot \frac{1}{\ln 2} = \frac{1}{x\ln 2}$

e. $f(x) = {}^3\log(5x-6)$

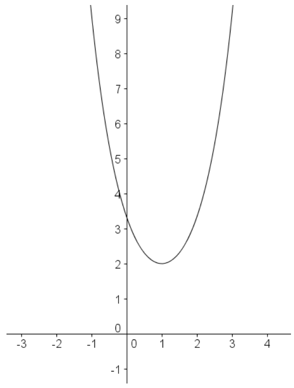
$f'(x) = \frac{1}{5x-6} \cdot 5 \cdot \frac{1}{\ln 3} = \frac{5}{(5x-6)\cdot \ln 3}$

f. $f(x) = \ln(3x^2 + 3)$

$f'(x) = \frac{1}{3x^2+3} \cdot 6x = \frac{6x}{3x^2+3} = \frac{2x}{x^2+1}$

Opgave 16:

a. $f(x) = 3^{x-1} + 3^{-x+1}$



$f'(x) = 3^{x-1} \cdot \ln 3 + 3^{-x+1} \cdot -1 \cdot \ln 3$

$= (3^{x-1} - 3^{-x+1}) \cdot \ln 3 = 0$

$3^{x-1} - 3^{-x+1} = 0$

$3^{x-1} = 3^{-x+1}$

$x - 1 = -x + 1$

$2x = 2$

$x = 1$

$y = 3^0 + 3^0 = 2$

$B_f = [2, \rightarrow\rangle$

b. $f'(x) = (3^{x-1} - 3^{-x+1}) \cdot \ln 3 = \frac{8}{3} \ln 3$

$3^{x-1} - 3^{-x+1} = \frac{8}{3}$

$3^{-1} \cdot 3^x - 3^1 \cdot 3^{-x} = \frac{8}{3}$

$\frac{1}{3} \cdot 3^x - 3 \cdot \frac{1}{3^x} = \frac{8}{3}$

neem $p = 3^x$ dan $\frac{1}{3} p - 3 \cdot \frac{1}{p} = \frac{8}{3}$

$p - 9 \cdot \frac{1}{p} = 8$

$p^2 - 9 = 8p$

$p^2 - 8p - 9 = 0$

$(p - 9)(p + 1) = 0$

$p = 9 \quad \vee \quad p = -1$

$3^x = 9 \quad \vee \quad 3^x = -1$

$x = 2$ k.n.

$y = 3 + 3^{-1} = 3 + \frac{1}{3} = 3\frac{1}{3}$

dus $(2, 3\frac{1}{3})$

Opgave 17:

a. $f(x) = \frac{\ln x}{x} = 0$

$\ln x = 0$

$x = 1$

$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$

$f'(1) = 1$

$y = x + b$ door $(1, 0)$

$0 = 1 + b$

$b = -1$

$y = x - 1$

b. $f'(x) = \frac{1 - \ln x}{x^2} = 0$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

$y = \frac{1}{e}$

$B_f = \langle \leftarrow, \frac{1}{e}]$